# Math 115B Homework 8

1) Express each rational number as a finite (simple) continued fraction.

a) $\frac{43}{30}$

b) $\frac{30}{43}$

c) $\frac{55}{34}$

d) $\frac{64}{391}$

2) Express each continued fraction as a rational number $\frac{a}{b}$.

a) [4,3,2,1]

b) [0,4,3,2,1]

c) [1,1,1,1,1,1]

d) [5,4,6,3,7]

3) Prove that every rational number is expressible as a finite simple continued fraction in exactly two ways.

4) Find all the convergents for the continued fractions in problem 2. Verify that their ordering satisfies Theorem 12.11.

5) Express each real number as an infinite simple continued fraction.

a) $\sqrt{7}$

b) $\sqrt{11}$

c) $\sqrt{13}$

d) $\sqrt{17}$

6) Find the first five convergents of the simple continued fraction expansion of each real number.

a) $\sqrt[3]{2}$

b) $\pi$

c) $e^2$

d) $\pi e$

7) Let $\alpha$ be a real irrational number, and let $p_i/q_i$ for $i - 0, 1, 2, \ldots$ be the convergents of the

infinite continued fraction expansion of $\alpha$. Prove that

$$\left|\alpha - \frac{p_i}{q_i}\right| < \frac{1}{q_i^2}$$

8) Find the quadratic irrational number associated with the continued fraction expansion $[1, 2, \overline{3, 4}]$.

9) Let $n$ be a positive integer. Prove that the period length of the infinite continued fraction expansion of $\sqrt{n}$ is 1 if and only if $n = m^2 + 1$ for some integer $m$.

10) How difficult was this homework? How long did it take?