

Math 21A, Practice problems for the Final

- 1) a) Write down the definition of a limit.
b) Prove using the definition that $\lim_{x \rightarrow 3} x^2 - 4 = 5$.
c) Prove that the function e^{x^2-4} is continuous at $x = 3$.
- 2) a) Find $\lim_{x \rightarrow 0} \frac{\sin 2x \cos x}{x}$. Justify every step!
b) Find $\lim_{x \rightarrow 1} \frac{\sqrt{x^3-2}}{2e^x}$. Justify every step!
c) Use the squeeze/sandwich theorem to find $\lim_{x \rightarrow 0} x \cos \frac{1}{x}$.
- 3) Find all asymptotes for the following functions, and justify your answers:
a) $\frac{x^2-3}{3x^2+6x+3}$
b) $\frac{x^2-1}{x^2+3x+2}$
c) $\frac{4x^4-3x+7}{x^3+1}$
d) $\sin \frac{1}{x}$.
- 4) Use the definition of derivative to show that $(\frac{1}{x^2+1})' = -\frac{2x}{(x^2+1)^2}$.
- 5) A light is on the top of a 12 foot tall pole and a 5.5 foot tall person is walking away from the pole at a rate of $2ft/sec$.
a) At what rate is the tip of the shadow of the person moving away from the *pole* when the person is 25 feet from the pole?
b) At what rate is the tip of the shadow moving away from the *person* when the person is 25 feet from the pole?
- 6) a) Find the first and second derivatives of $\sin^{-1} \ln x$ where $1/e < x < e$. Justify every step.
b) Find the seventh derivative of $x^{22} + 6x^6 + 3x^2 + 4x + 11$. Justify your answer.
- 7) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $\ln(x+y) = y^2$.
- 8) Find all the critical points, local/absolute maxima and minima, and inflection points of the following functions over the given intervals. Determine where the functions are concave up and down, and sketch the function over the interval provided.
a) $x^4 - 3x^2 + 2$ over $[-5, 5]$
b) $\ln(\sin x)$ over $(0, \pi)$

c) $x^{9/7} + 3$ over $[-3, 3]$

9) A company wants to construct a box whose base length is 3 times the base width. The material used to build the top and bottom costs 10 dollars per square foot, and the material used to build the sides costs 6 dollars per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box. Show your work.

10) Compute the following limits.

a) $\lim_{x \rightarrow 1} \frac{x \ln x}{x^2 - 1}$

b) $\lim_{x \rightarrow -4} \frac{\sin \pi x}{x^2 - 16}$

c) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{1 - \cos 3x}$

d) $\lim_{x \rightarrow \infty} (2x)^{1/x}$