Math 21A, Useful Final formulas and facts

LIMIT LAWS: Let $f(x)$, $g(x)$ be functions, let L, M, k be real numbers, let c be either a real number or ∞ or $-\infty$, and let n be a positive whole number. Suppose $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$.

- Sum law: $\lim_{x\to c}(f(x) + g(x)) = L + M$
- Difference law: $\lim_{x\to c}(f(x) - g(x)) = L - M$
- Product law: $\lim_{x\to c} f(x)g(x) = L \cdot M$
- Constant multiple law: $\lim_{x\to c}(k \cdot f(x)) = k \cdot L$
- Exponent law: $\lim_{x\to c}(f(x))^n = L^n$
- Quotient law: $\lim_{x\to c}(f(x)/g(x)) = L/M$ if $M \neq 0$
- Root law: $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$. Here, if n is even we must have $L > 0$ for this to be true.

SQUEEZE/SANDWICH THEOREM: Let $f(x) \leq h(x) \leq g(x)$ for all x in the open interval (a, b), where a, b are real numbers, or ∞, or $-\infty$. Suppose $a < c < b$ (here we allow c to be ∞ or $-\infty$ in the case where a or b is ∞ or $-\infty$). Finally, suppose

$$\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = L$$

where L is a real number. Then

$$\lim_{x\to c} h(x) = L.$$

In a nutshell, if f and g squeeze h between them and have the same limit at c, h is forced to have that same limit at c as well.

TWO USEFUL LIMITS:

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

$$\lim_{\theta\to 0} \frac{1 - \cos\theta}{\theta} = 0$$

DIFFERENTIATION RULES: Disclaimer: we do not include the various conditions that need to hold in order for these rules to be true. These are meant just as a reminder of what the rules are.

- Sum rule: $(f+g)'(x) = f'(x) + g'(x)$
- Constant multiple rule: $(c \cdot f)'(x) = c \cdot f'(x)$ if c is a constant
- Product rule: $(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$
- Quotient rule: $(f/g)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$
- Chain rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$
- Inverse function rule: $(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$

USEFUL DERIVATIVES:

- $(x^n)' = nx^{n-1}$ if n is a real number
- $(a^x)' = \ln a \cdot a^x$
- $(\ln x)' = \frac{1}{x}$
- $(\cos x)' = -\sin x$
- $(\sin x)' = \cos x$. Other trigonometric derivatives can be derived using quotient rule and the derivatives of cosine and sine.
- $(\sin^{-1}(x))' = \frac{1}{\sqrt{1-x^2}}$
- $(\cos^{-1}(x))' = -\frac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1}(x))' = \frac{1}{1+x^2}$

Trigonometric facts:

- $\sin 0 = \cos \frac{\pi}{2} = 0$, $\sin \frac{\pi}{6} = \cos \frac{\pi}{3} = \frac{1}{2}$, $\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin \frac{\pi}{3} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin \frac{\pi}{2} = \cos 0 = 1$
- The range of $\sin^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. The range of $\cos^{-1}(x)$ is $[0, \pi]$. The range of $\tan^{-1}(x)$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Idea behind using linearization to approximate a function close to a point x, at $x + \Delta x$:

$$\frac{\Delta y}{\Delta x} \approx f'(x)$$

Mean Value Theorem:

Suppose $f(x)$ is differentiable on (a, b). Then there is a point c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rough statement of L'Hopital's Rule (not all conditions are included):

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ or $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Another useful theorem is that

$$\lim_{x\to a} f(x) = e^{\lim_{x\to a} \ln f(x)}$$

if $\lim_{x\to a} \ln f(x)$ exists.