Final Dec 5th.

Next week OH:

- Tues 3-5 PM

  Thurs 2-4 PM

Review Wednesday 6-7:30 PM

- can email me questions between Thurs, Saturday, and I will post responses.

- Dec 3rd, 4th
  - ↳ TA's will be in 1147 MSB 10AM-3:30PM. to help.
  - ↳ $10 per person per day.

---

local min at a:

$f'(a) = 0$ or DNE

f'(left of a) ≤ 0

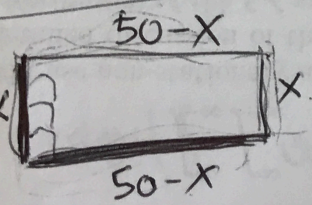
f'(right of a) ≥ 0

local max at a:

- $f'(a) = 0$ or DNE
- f'(left of a) ≥ 0
- f'(right of a) ≤ 0

Recall the problem: (rectangular)

Candy shop makes candy boxes with perimeter 100 inches, put 1 inch square chocolates inside. Optimize number of chocolates per box



← want to optimize area

$A = x(50-x) = 50x - x^2$

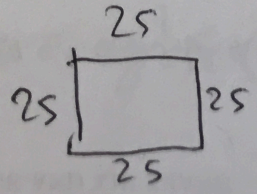
$0 \leq x \leq 49$

$\frac{dA}{dx} = 50 - 2x \stackrel{?}{=} 0$

↳ when $2x = 50$

$\Rightarrow x = 25$

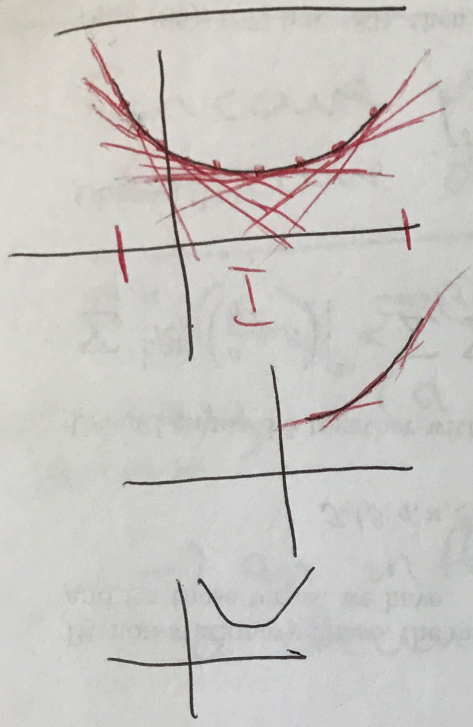
to the left of 25, $50-2x$ is positive, to the right it is negative



$\Rightarrow x = 25$ gives local max.

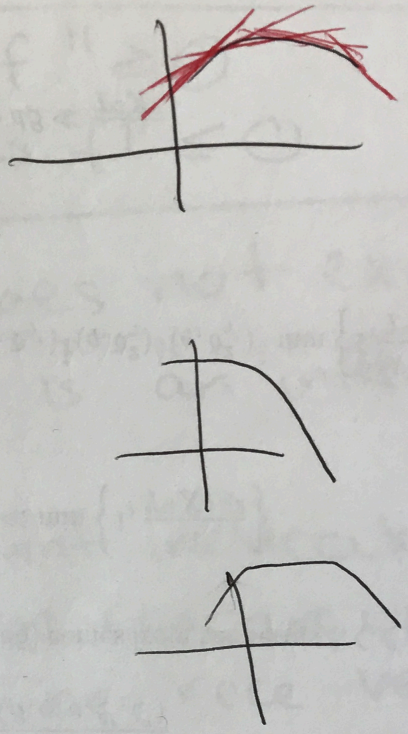
Second Derivatives and Concavity

Concave up



f is concave up on an interval I when f' is increasing on I.

Concave down



f is concave down on an interval I when f' is decreasing on I.

to tell when g is increasing,
check when is $g' \geq 0$
to tell ——————— decreasing,
check ——— $g' \leq 0$

So to tell when $g = f'$ is increasing or decreasing, check when $g' = f''$ ≥ 0 or ≤ 0.

Concave up $\iff f'' \geq 0$

Concave down $\iff f'' \leq 0$.

---

if $f''(a) = 0$ or does not exist, we say $(a, f(a))$ is an inflection point.

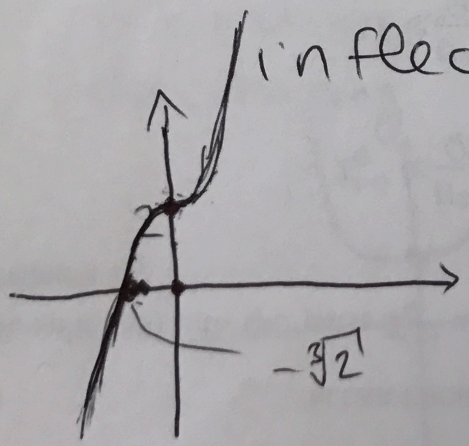
↳ an inflection point indicates a change from concave up to down or vice versa.

EX: $f(x) = x^3 + 2$ ⇝ find where this is concave up/down.

$f'(x) = 3x^2 \rightsquigarrow = 0$ at $x=0$; $3x^2 \geq 0$ for all x

$f''(x) = 6x \rightsquigarrow 6x \geq 0$ when $x \geq 0$

$6x \leq 0$ when $x \leq 0$



inflection point at $x=0$

$x^3 + 2 = 0$?

$\rightsquigarrow x^3 = -2 \rightsquigarrow x = \sqrt[3]{-2}$

EX $f(x) = x^{7/5}$ $\frac{7}{5}(x^{\frac{1}{5}})^2$

$f'(x) = \frac{7}{5}x^{2/5}$ $f''(x) = \frac{14}{25} \cdot x^{-\frac{3}{5}}$

$= \frac{14}{25x^{3/5}}$

critical point: $x = 0$

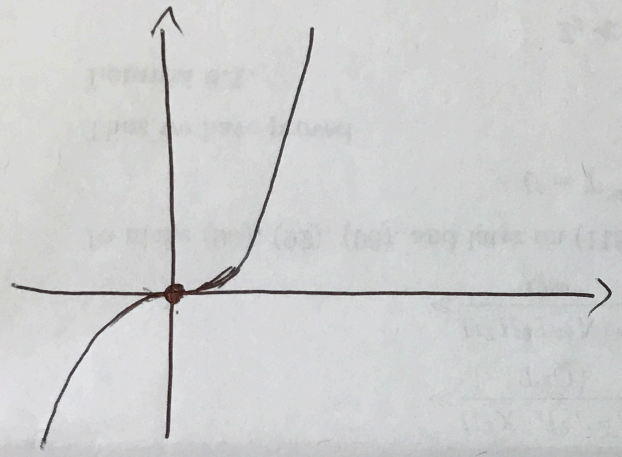
increasing everywhere.

inflection points: $x = 0$

$f''(0)$ DNE

Concave up for $x \geq 0$

Concave down for $x \leq 0$



EX: $y = 3x^4 + 2$

$y' = 12x^3$ $y'' = 36x^2$

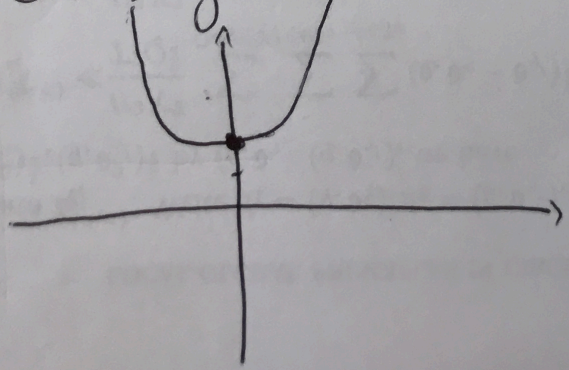
critical points: $x = 0$

increasing $x \geq 0$

decreasing $x \leq 0$.

inflection pt: $x = 0$

Concave up everywhere



Translate from before:

if $f'(a) = 0$, $f''(a) < 0$ $\rightsquigarrow$ local max at a

if $f'(a) = 0$, $f''(a) > 0$ $\rightsquigarrow$ local min at a

if $f'(a) = 0$, $f''(a) = 0$ $\rightarrow$ can't tell.