Math 21A, Practice Midterm 1

1) A ball that is dropped to fall freely travels about $y = 16t^2$ feet after t seconds.

a) Find the average speed of the ball between the 5th and 7th second.

b) Find the average speed of the ball between the 5th and 6th second.

c) Using the idea of average speeds, how would you estimate the speed of the ball at time $t = 5$ seconds? Write down the limit that would allow you to find this speed and compute what the limit is.

d) What does the value of the limit in part (c) correspond to in the graph of $y = 16t^2$?

2) a) What is the formal definition of the limit of a function f(x) at a point c when it exists? In other words, define what is meant by $\lim_{x\to c} f(x) = L$.

b) Coming back to the scenario in problem (1), find a time interval in which the falling ball has fallen a distance that is within 0.2 feet of 32 feet.

c) Using only the formal definition from part (a), show that $\lim_{x\to 2}(x^3 - 5) = 3$.

3) a) What is the definition of the statement "a function $f(x)$ is continuous at $x = c$"?

b) True or false: if two functions, $f(x)$ and $g(x)$ are both continuous at $x = 10$, then $f \circ g$ is continuous at $x = 10$. Justify your answer.

c) Is the function $\sin(e^{\frac{x^3-3x^2+2}{4x^5+19}})$ continuous at $x = 1$? Justify your answer, in which you may assume that the sine and exponential functions are continuous everywhere.

d) Compute

$$\lim_{x\to 1^-} (\sin(e^{\frac{x^3-3x^2+2}{4x^5+19}}) + x^4 - 2)$$

4) a) Find

$$\lim_{x\to 0^+} \frac{\sin 3x}{1 - \cos x}$$

It may be useful to know the half angle formula for sine: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$.

b) Find

$$\lim_{x\to 0^-} \frac{\sin 3x}{1 - \cos x}$$

5) Find (write down equations of) all the horizontal, vertical, and oblique asymptotes for the following functions. Justify your answers.

a) $f(x) = \frac{\sin x}{x}$

b) $f(x) = \sin \frac{1}{x}$

c) $f(x) = \frac{4x^5 - 6x^3 + 1}{x^5 - 1}$

d) $f(x) = \frac{5x^3 - 6x + 6}{x^2 - 1}$