Math 21A, Practice Midterm 2

1) a) Write down the limit definition of $f'(a)$, where $f(x)$ is differentiable at a.

b) Show that the derivative of $\frac{2}{x}$ is $-\frac{2}{x^2}$ using the definition in part (a).

c) Find the value of the following limit:

$$\lim_{h \to 0} \frac{\ln(3+h) - \ln 3}{h}.$$

Justify your answer.

d) Use linearization to approximate $\tan(\frac{\pi}{3} + 0.1)$. You do not need to give a decimal expansion expression of your answer.

2) True or false: if an everywhere differentiable function $f(x)$ has a horizontal asymptote $y = L$ then $f'(x)$ has a horizontal asymptote $y = 0$.

3) a) Use differentiation laws (not logarithmic differentiation) to find the derivative of

$$\frac{(\sec^2 x)(x+3)^2}{e^x}.$$

Be sure to specify which laws you are using at every step.

b) Use logarithmic differentiation to evaluate the derivative in part (a).

4) a) Show that

$$(\cos^{-1}(x))' = -\frac{1}{\sqrt{1-x^2}}$$

using only what you know about the derivative of $\sin^{-}1(x)$ and the fact that $\cos^{-1}(x) = \pi/2 - \sin^{-1}(x)$.

b) Suppose $f(x) = x^3$ and let $g = f^{-1}$, the inverse of f. Compute $g'(8)$ and $g''(8)$.

c) Compute the derivative of $\frac{\sin^{-1}(x)}{\cos^{-1}(x)}$.

5) a) Let $x^2 - \sec(x+y) = y^2 + 2$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b) Let $y^3 = x^2 + 2y$. Find $\frac{d^3y}{dx^3}$.

6) A spherical iron ball 8 inches in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 10 cubic inches per minute, how fast is the thickness of the ice decreasing when it is 2 inches thick? How fast is the outer surface area of the ice decreasing? Note: volume of a sphere is $4\pi r^3/3$ and surface area is $4\pi r^2$.