Canvas → Files

↳ solutions to some hw problems

↳ slides from class

↳ answers to evens

Practice Midterm

1) a) average speed between $t=5$ and 7

$$\frac{16\cdot 7^2 - 16\cdot 5^2}{7-5} = \frac{16(7^2-5^2)}{2} =$$

$$= 8\cdot 24 = 192 \text{ ft/s}$$

b) avg speed between $t=5$ and 6

$$\frac{16\cdot 6^2 - 16\cdot 5^2}{6-5} = \frac{16(36-25)}{1} = 176 \tfrac{ft}{s}$$

c) Consider the avg speed between $t=5$ and 5.001

When can you just plug in?

↳ when is $\lim_{x \to c} f(x) = f(c)$

↳ 1) polynomials $f(x) = P(x)$

2) $\frac{\text{polynomial}}{\text{polynomial}} \rightsquigarrow \frac{P(x)}{Q(x)}$ as long as $Q(c) \neq 0$

3) functions that are continuous at c.

↳ $\cos x$, $\sin x$, e^x are continuous at all x

↳ "inverse functions of cts functions are cts"

"

$\ln x$ is continuous on its domain

4) If $f(x)$ is cts at c, and $g(x)$ is cts at $f(c)$ then $g \circ f$ is cts at c.

Ex: $\lim_{x \to 5} \ln x = \ln 5$

e^x is cts everywhere so $\ln x$ (which is the inverse of e^x) is continuous on its whole domain.

c) $\lim_{h \to 0} \dfrac{16(5+h)^2 - 16\cdot 5^2}{\cancel{5}+h-\cancel{5}}$

$= \lim_{h \to 0} \dfrac{16(25+10h+h^2) - 16\cdot 25}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{16\cdot 25} + 160\cancel{h} + 16h^{\cancel{2}} - \cancel{16\cdot 25}}{\cancel{h}}$ $160+16h$

$= 160$ ft/s

polynomial

d) slope of tangent line to $y=16t^2$ at $(5, 400)$.

2) a) $\lim_{x \to c} f(x) = L$

NOTE: you can do

Defn: for any $0.1 > \varepsilon > 0$ there exists a number $\delta > 0$ such that

if $0 < |x - c| < \delta$

then $|f(x) - L| < \varepsilon$

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b) find time interval in which the ball in problem 1 ~~is~~ has fallen between 31.8 ft and 32.2 ft

for which t is $|16t^2 - 32| < 0.2$

$$-0.2 < 16t^2 - 32 < 0.2$$
$$+32 \quad +32 \quad +32$$

$$31.8 < 16t^2 < 32.2$$

$$\frac{31.8}{16} < t^2 < \frac{32.2}{16}$$

sort of like proving $\lim_{t \to \sqrt{2}} 16t^2 = 32$

$\frac{32 - \varepsilon}{16}$

$$\boxed{\sqrt{\frac{31.8}{16}} < t < \sqrt{\frac{32.2}{16}}}$$

can write "For ε small enough"

c) Show $\lim_{x \to 2} (x^3 - 5) = 3$

For any $\varepsilon > 0$, find a $\delta > 0$ such that if

$$0 < |x - 2| < \delta$$

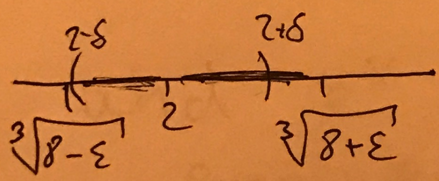
then $|x^3 - 5 - 3| < \varepsilon$

$$-\varepsilon < x^3 - 8 < \varepsilon$$

$$8 - \varepsilon < x^3 < 8 + \varepsilon$$

cube root

$$\sqrt[3]{8-\varepsilon} < x < \sqrt[3]{8+\varepsilon}$$



$$\delta = \min\left(\sqrt[3]{8+\varepsilon} - 2,\ 2 - \sqrt[3]{8-\varepsilon}\right)$$

3) a) $\lim_{x \to c} f(x) = f(c)$

$\hookrightarrow \lim_{h \to 0} f(\cancel{c}+h) = f(\cancel{c})$

b) False:

the correct statement is

if $g(x)$ is continuous at $x=10$ and $f(x)$ is continuous at $g(10)$ then $f \circ g$ is continuous at $x=10$

c) Yes. Why: because

$\frac{x^3 - 3x^2 + 2}{4x^5 + 19}$ is cts at 1

since this is $\frac{\text{polynomial}}{\text{polynomial}}$ and

$4 \cdot 1^5 + 19 \neq 0$

and e^x, $\sin x$ are continuous everywhere. By composition of continuous functions theorem, $\sin\left(e^{\frac{x^3-3x^2+2}{4x^5+19}}\right)$ is continuous at 1.

d) compute

$$\lim_{x \to 1^-} \left(\sin\left(e^{\frac{x^3-3x^2+2}{4x^5+19}}\right) + \underbrace{x^4-2}_{\text{polynomial}} \right)$$

$$= \sin\left(e^{\frac{1-3+2}{4+19}}\right) + 1 - 2$$

in part c: is continuous at 1,

sum limit law, polynomial law

4) a) Compute ← given $\sin^2 x = \frac{1-\cos 2x}{2}$ ⇝ $\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$

$$\lim_{x\to 0^+} \frac{\sin 3x}{1-\cos x}$$

$$= \lim_{x\to 0^+} \frac{\sin 3x}{2\sin^2 \frac{x}{2}} \cdot \frac{x^2}{x^2}$$

(Recall: $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 = \lim_{\theta\to 0} \frac{\theta}{\sin\theta}$)

$$\left(\frac{x/2}{\frac{1}{2}\sin\frac{x}{2}}\right)^2$$

$$= \lim_{x\to 0^+} \boxed{\underbrace{\frac{3\sin 3x}{3x}}_{\to 3} \cdot \underbrace{\frac{1}{x}}_{+\infty} \cdot \frac{1}{2} \cdot \underbrace{\boxed{\frac{x^2}{\sin^2\frac{x}{2}}}}_{\to 4}}$$

$$= \infty$$

b) $\lim_{x\to 0^-} \frac{\sin 3x}{1-\cos x} = \lim_{x\to 0^-} \qquad = -\infty$

since $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$f(x)$ has a horizontal asymptote $y = L$ if $\lim_{x \to \infty} f(x) = L$

or

$\lim_{x \to -\infty} f(x) = L$

$f(x)$ has a vertical asymptote $x = c$ if

$\lim_{x \to c^{\pm}} f(x) = \pm\infty$

5) a) $\lim_{x \to \pm\infty} \frac{\sin x}{x} = 0$

$\Rightarrow$ horiz. asymptote $y = 0$.

no vert. asymptote.
no oblique as.

b) $f(x) = \sin \frac{1}{x}$

$\lim_{x \to \pm\infty} \sin \frac{1}{x} = 0$

$\Rightarrow$ horiz. as. at $y = 0$

no vert asymptotes.

c) $$\lim_{x \to \pm\infty} \frac{4x^5 - 6x^3 + 1}{x^5 - 1} = 4$$

$\rightsquigarrow$ horiz as. is $y = 4$

$\rightsquigarrow$ vert as. at $x = 1$

since $1^5 - 1 = 0$.

$\llcorner$, and so $\displaystyle\lim_{x \to 1^+} \frac{4x^5 - 6x^3 + 1}{x^5 - 1} = \infty$

d) $$f(x) = \frac{5x^3 - 6x + 6}{x^2 - 1}$$

no horiz. as.

vert. asymptotes $x = \pm 1$

oblique asymptote:

$$\begin{array}{r|l} & 5x \\ \hline x^2 - 1 & 5x^3 - 6x + 6 \\ & -\ 5x^3 + 5x \\ \hline & \quad -x + 6 \end{array}$$

$y = 5x$ is the oblique as.

(why: $f(x) = 5x + \dfrac{-x + 6}{x^2 - 1}$)