Math 22 A: Homework 1

1. Let
$$\overline{x} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \overline{y} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$
Draw in the plane the vectors $\overline{x}$, $\overline{y}$, $\overline{x} + \overline{y}$, $2 \cdot \overline{x} - 3 \cdot \overline{y}$.

2. Let
$$\overline{x} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \quad \overline{y} = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$$
Let $\overline{a}$ be an arbitrary linear combination of $\overline{x}$ and $\overline{y}$, hence there are scalars c_1, c_2 with
$$\overline{a} = c_1 \cdot \overline{x} + c_2 \cdot \overline{y}$$
Write
$$\overline{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$
Show that, whatever the values of c_1 and c_2 are, one has
$$a_1 + a_2 + a_3 = 0$$
Use this to find a vector that is not a linear combination of $\overline{x}$ and $\overline{y}$.

3. Find all pairs (c_1, c_2) of scalars c_1 and c_2 such that
$$c_1 \cdot \begin{bmatrix} -5 \\ 2 \end{bmatrix} + c_2 \cdot \begin{bmatrix} 10 \\ -4 \end{bmatrix} = \overline{0}$$

4. Let
$$\overline{x} = \begin{bmatrix} \pi \\ 1 \\ 2 \end{bmatrix}, \quad \overline{y} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \quad \overline{z} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$
Calculate the inner product $\overline{x} \cdot \overline{y}$ and $(\overline{x} + \overline{y}) \cdot \overline{z}$

5. For $\overline{y}$ and $\overline{z}$ as in the previous problem, verify that
$$|\overline{y} \cdot \overline{z}| \leq ||\overline{y}|| \cdot ||\overline{z}||$$

6.

Recall that a unit vector is a vector of length 1. Find a unit vector in the direction of

$$\overline{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

7. Let $\overline{x}$ and $\overline{y}$ be unit vectors. Hence $\overline{x} \cdot \overline{x} = \overline{y} \cdot \overline{y} = 1$. Calculate the inner products $\overline{x} \cdot (-\overline{x})$ and $(\overline{x} + \overline{y}) \cdot (\overline{x} - \overline{y})$.
8. Find non-zero vectors $\overline{x}$ and $\overline{y}$ that are both orthogonal to

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and orthogonal to each other.

9. Find all vectors orthogonal to

$$\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$