Math 22 A: Homework 2

1. Calculate the following matrix vector product in the two ways that we discussed in class: As a linear combination of the columns and in terms of dot products of the rows.

$$\begin{bmatrix} 2 & 3 & 0 & 5 \\ -3 & 2 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

2. In class we showed that the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

corresponds to the function on vectors given by

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ -y \end{bmatrix}$$

Now find the matrix that corresponds to the function

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} y \\ x \end{bmatrix}$$

3. Decide if there is a three by three matrix A such that

$$A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z^2 \\ x \\ y \end{bmatrix}$$

for all x, y, z in $\mathbb{R}$.

4. Decide if there is a three by three matrix A such that

$$A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 1 \\ z \end{bmatrix}$$

for all x, y, z in $\mathbb{R}$.

5. Find the three by three matrix A such that

$$A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y \\ y \\ z \end{bmatrix}$$

for all x, y, z in $\mathbb{R}$.

6.

Find the two by two matrix A such that

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}$$

Give the geometric interpretation of this function.

7. We mentioned in class that if A is an $n \times n$ matrix and $\overline{x}$ and $\overline{y}$ are n-component vectors and c is a scalar then

$$A \cdot (\overline{x} + \overline{y}) = A \cdot \overline{x} + A \cdot \overline{y}$$

$$A \cdot (c \cdot \overline{x}) = c \cdot (A \cdot \overline{x})$$

Verify these two properties by direct computation in the case where

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -5 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

and

$$\overline{x} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} \text{ and } \overline{y} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \text{ and } c = 4$$