Math 22 A: Homework 4

1. Find the inverse of the following matrices

   (a)
   $$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ -4 & 0 & 0 & 0 \end{bmatrix}$$

   (c)
   $$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

2. We call a matrix invertible if it has an inverse and a matrix is called singular if it has no inverse. The sum $A + B$ of two square matrices A and B is defined component wise, hence for example
   $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} \pi & -1 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 1+\pi & 1 \\ 3 & 11 \end{bmatrix}$$

   (a) Give an example of invertible matrices A and B with $A + B$ singular.

   (b) Give an example of singular matrices A and B such that $A + B$ is invertible.

3. Show that a matrix of the form
   $$\begin{bmatrix} * & * & 0 & * \\ * & * & 0 & * \\ * & * & 0 & * \\ * & * & 0 & * \end{bmatrix}$$
   has no inverse.

4. For
   $$A = \begin{bmatrix} 2 & 2 & 1 \\ 4 & 0 & -3 \\ -10 & 1 & 1 \end{bmatrix}$$
   find a factorization $A = L \cdot U$ where L is lower triangular and U is upper triangular.

5.

Show that

$$A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$$

has no $A = L{\cdot}U$ factorization where L is lower triangular and U is upper triangular: Suppose for contradiction that

$$A = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \cdot \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$

and show that there are no choices of a, b, c, x, y, z such that this equation holds.

6. Show that

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

has two different factorizations of the form $A = L \cdot U$ where L is lower triangular with 1's on the diagonal and U is upper triangular.

7. Recall that a subset V of $\mathbb{R}^n$ is a subspace of two conditions hold:

Condition (I): For all $\overline{x}, \overline{y}$ in V one has $\overline{x} + \overline{y}$ in V.

Condition (II): For all $\overline{x}$ in V and all scalars c one has $c \cdot \overline{x}$ in V.

(a) Find a subset V of $\mathbb{R}^2$ such that condition (I) holds but condition (II) fails.

(b) Find a subset V of $\mathbb{R}^2$ such that condition (II) holds but condition (I) fails.

8. Decide, with justification if the following subsets of $\mathbb{R}^3$ are in fact subspaces of $\mathbb{R}^3$:

(a)

$$V = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{with} \quad b_1 \cdot b_2 + b_3 = 0 \right\}$$

(b)

$$V = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{with} \quad 2 \cdot b_1 - \pi \cdot b_2 + b_3 = 0 \right\}$$

(c)

$$V = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{with} \quad b_1 \cdot b_3 \leq 10 \right\}$$