

Math 22 A: Homework 5

- Recall that the column space $C(A)$ of a matrix A is the set of linear combinations of the columns of A . Now let A and B be two $n \times n$ matrices.
 - Show that $C(A \cdot B)$ is contained in $C(A)$.
 - Give an example (say for $n = 2$) where $C(A \cdot B) = C(A)$ and an example where $C(A \cdot B)$ is not equal to $C(A)$.
- Give an example of a 3×3 matrix A with $C(A) = \mathbb{R}^3$.
 - Give an example of a 3×3 matrix A with $C(A)$ a line.
- Show that for two matrices A and B one has that $\text{Null}(B)$ is contained in $\text{Null}(A \cdot B)$.
- Suppose $A \cdot B = 0$. Show that this implies that $C(B)$ is contained in $\text{Null}(A)$.

5. Calculate the rank of

$$\begin{bmatrix} 1 & 2 & 5 & -1 \\ 0 & 1 & 2 & -1 \\ -1 & 1 & 1 & -2 \\ 2 & 1 & 4 & 1 \end{bmatrix}$$

6. Find the reduced row echelon form of the following matrices:

$$A = \begin{bmatrix} 2 & 0 & 10 \\ -2 & 1 & -9 \\ 4 & 1 & 21 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & -5 \\ 3 & 2 & 1 \end{bmatrix}$$

7. Describe all solutions to:

(a)

$$\begin{bmatrix} 2 & 0 & 10 \\ -2 & 1 & -9 \\ 4 & 1 & 21 \end{bmatrix} \cdot \bar{x} = \begin{bmatrix} 2 \\ 3 \\ 9 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 0 & 10 \\ -2 & 1 & -9 \\ 4 & 1 & 21 \end{bmatrix} \cdot \bar{x} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix} \cdot \bar{x} = \begin{bmatrix} 0 \\ 12 \\ 8 \end{bmatrix}$$