Math 22 A: Homework 6

1. Decide with justification if the following sets of vectors are linearly independent.

   (i) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \right\}$

   (ii) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

   (iii) $\left\{ \begin{bmatrix} -2 \\ 2 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

2. Find a basis of Null(A), C(A), and C(A^{T}) for each of the following matrices:

   (i) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (ii) $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

   (iii) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   (iv) $A = \begin{bmatrix} 2 & -4 & 1 & -3 \\ 3 & -6 & 1 & -5 \\ 0 & 0 & 2 & 2 \\ -5 & 10 & 3 & 13 \end{bmatrix}$

3. Suppose V is a subspace of $\mathbb{R}^n$ and suppose $\{v_1, v_2, v_3\}$ is a basis of V. Decide if the following sets of vectors are a basis for V:

   (i) $\{v_2, v_1 - 5v_3, 2v_3\}$

   (ii) $\{v_2, v_1 - 5v_3, 2v_3, 3v_2 + 7v_3 - v_1\}$

   (iii) $\{2v_2 - v_3, v_1\}$

4. Suppose that two vectors $\overline{x}$ and $\overline{y}$ in $\mathbb{R}^n$ are orthogonal, meaning their inner product satisfies $\overline{x} \cdot \overline{y} = 0$. Show that if both vectors are assumed to be non-zero, then $\{\overline{x}, \overline{y}\}$ are linearly independent. Show that this linear independence fails if at least one of the vectors is the zero vector.