Math 22 A: Homework 7

1. (a) Find the matrix P describing the projection from $\mathbb{R}^3$ onto the subspace

$$V = \text{span}\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} \}$$

   (b) Find the projections

$$\text{proj}_V(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}) \text{ and } \text{proj}_V(\begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix})$$

   (c) Find P^3 (ideally with as little work as possible).

2. (a) Find the projection matrix P describing the projection of $\mathbb{R}^4$ onto

$$V = \text{span}\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 1 \\ 1 \end{bmatrix} \}$$

   (b) Calculate rank(P) by bringing P to reduced row echelon form. Can you give a geometric argument for the answer you obtained for the rank?

3. Find (ideally with as little work as possible) the projection matrix P describing the projection from $\mathbb{R}^5$ to the column space $C(A)$ of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 6 & 7 & 8 \\ 0 & 0 & 1 & 9 & 10 \\ 0 & 0 & 0 & 1 & 11 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4. Find the least squares solution of

$$\begin{bmatrix} 1 & 5 \\ 2 & 0 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 1 \\ 1 \end{bmatrix}$$

5. (a) Show that $\{v_1, v_2, v_3\}$ are linearly independent, where

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \text{ and } v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

   Hence $\{v_1, v_2, v_3\}$ is a basis of $V = \text{span}\{v_1, v_2, v_3\}$.

   (b) Find an orthonormal basis of V.