Math 250A: Reading and Concepts for the week of October 11th and October 18th

- Finishing semidirect products. Review the notion of center and class equation.
- Group actions (we won't prove thee orbit stabilizer lemma, and if you have never seen it before you may want to read its proof)
- Sylow theorems, proof of first Sylow theorem and a useful tool to prove the rest. Think about or look up proof of the following: an abelian group whose order is divisible by a prime p has an element of order p. I will assume you are comfortable with the conjugation action.
- Using Sylow theorems in examples, motivating the study of fields and Galois theory.

General reading note: at this point we are soon transitioning into the study of fields: specifically, roots of polynomials, algebraic extensions, finite fields, and Galois theory to name some topics. I will assume that you are very familiar (and may do a review lecture) with the following concepts/results from the get-go (F always denotes a field):

- fields, field characteristic, field extension, simple field extension, degree of a field extension, finite extensions, algebraic vs. transcendental elements, minimal polynomial of an algebraic element,
- $F(\alpha) \cong F[x]/(p(x))$ where $p(x)$ is an irreducible polynomial in $F[x]$ and α is a root of $p(x)$; this tells us how to construct a field containing a zero of a given polynomial over F
- Irreducibility criteria for polynomials: see section 2.5
- A polynomial $p(x) \in F[x]$ can have at most $\deg(p)$ zeros in F
- A field has characteristic 0 or p for some prime p
- A finite multiplicative subgroup of a field is cyclic
- α is algebraic over F if and only if the degree of α over F is finite
- Finite extensions are algebraic
- Tower theorem: If E is a finite extension of F and F is a finite extension of K then $[E:K] = [E:F][F:K]$