Math 250A: some practice problems for the midterm

1. (a) Show that there is a nonabelian subgroup T of $S_3 \times \mathbb{Z}_4$ of order 12 generated by elements a, b such that $|a| = 6$, $a^3 = b^2$, and $ba = a^{-1}b$.

   (b) Show that any group of order 12 with generators a, b such that $|a| = 6$, $a^3 = b^2$, and $ba = a^{-1}b$ is isomorphic to T.

2. Classify up to isomorphism all groups of order 18.

3. Let G be a group such that $G = AB$ where A, B are abelian subgroups of G. Show that $[G, G] = [A, B]$ where

$$[A, B] := \langle aba^{-1}b^{-1} \mid a \in A, b \in B \rangle.$$

4. Show that if H, K are solvable subgroups of G such that $H \trianglelefteq G$, then HK is a solvable subgroup of G.

5. Prove the fundamental theorem of arithmetic (unique prime factorization) using the Jordan-Hölder theorem applied to $\mathbb{Z}_n$.