Math 250A: Reading and Concepts for Lectures 1-4

General reading note: If you are not comfortable with undergraduate material defining groups, cyclic groups, homomorphisms, and cosets, as well as some classical examples like S_n and D_{2n}, please read up on it. Also, for a summary of things you need to know about cyclic groups, see Arthur Ogus's nice concise notes on our course website. To refresh your comfort with permutation groups, it might help also to look over George Bergman's notes on the proof that A_n is simple for $n \geq 5$ (also posted on the course website).

The lectures this week will be planned roughly as follows (it may very well take longer to cover than planned):

- Lectures 1 and 2: Quotient groups and isomorphism theorems, a bit on exact sequences. I assume you know what the following are: groups, (normal) subgroups, quotient groups, index, homomorphism, image, kernel, the canonical morphism $G \to G/H$ where $H \trianglelefteq G$, centralizer and normalizer (I may remind you briefly what some of these are).

  It will be useful to think about the following exercises:
  - If K is any subgroup of G containing H and such that H is normal in K, then K is contained in N_H, the normalizer of H in G.
  - If K is a subgroup of N_H, then KH is a group and H is normal in KH.
  - The normalizer of H is the largest subgroup of G in which H is normal.
- Lectures 3 and 4: Composition series, beginning Jordan-Hölder theorem, abelian and cyclic towers (series). I assume you know what simple, abelian, and cyclic groups are. You should be comfortable with the isomorphism theorems covered last time, and should convince yourself of the following: If G_1, H_1 are normal subgroups of G, then G_1H_1 is a normal subgroup of G. Also, if H is a normal subgroup of G and K is a normal subgroup of G contained in H, then H/K is a normal subgroup of G/K. Finally, the intersection of normal subgroups is normal.