MATH 21C: Calculus 3
E. Kim: Summer 2006 Session 1
PRACTICE FINAL

NAME .....

I.D. NUMBER .....

With my signature, I certify that the work on this examination is wholly my own, and that: (1) I have not received any assistance from others, (2) I have not assisted anyone else, and (3) I have not used any reference material [books, notes, etc.] or tool [calculator, slide rule, pager, cell phone, laptop, etc.] not specifically authorized for this examination.

SIGNATURE .....

- All cell phones and pagers must be off during the test.
- Read directions carefully.
- You must show your work to earn credit: Even if you do not fully solve a problem, relevant work leading to a successful solution earns partial credit. A correct answer with no work shown can earn zero credit!! **SHOW ALL OF YOUR WORK**.
- Cross out irrelevant scratch work. Ensure that your answers are both thorough and clear: unclear answers are also liable for reduced credit!
- Do not begin until told to do so. In fairness to other students, you must return this exam at 9:40.

Page numbers are printed at the bottom.

INITIALS.....

1. [10 pts.] What is the area of the triangle with vertices $A = (-3, 8)$, $B = (4, 2)$, and $C = (-5, -6)$?

2a. [5 pts.] What is the derivative of the vector function $\mathbf{r}(t) = \langle \cos(3t), \ \ t, \ \ \sin(4t) \rangle$

2b. [5 pts.] State the Angle Formula for the angle θ between two vectors $\mathbf{u}$ and $\mathbf{v}$.

2c. [5 pts.] Compute the cross product $\mathbf{u} \times \mathbf{v}$ if $\mathbf{u} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 1\mathbf{k}$.

INITIALS.....

3. [15 pts. each] Find the gradient ∇f for each function below:

$f(x, y, z) = 2xy + y^2z^3 + x^{10} - z$

$f(x, y, z) = \cos(\sin y) + \sin(\cos x) + x^2y^2z^2$

$f(x, y, z) = \sin y \cos x + xye^{3y}$

INITIALS.....

4. [5 pts.] What is the length of the vector $\mathbf{w} = \langle \pi, \sqrt{2}, -80 \rangle$?

5. [10 pts.] Let $\mathbf{u} = \langle 1, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, -2, 3 \rangle$. There are two vectors of length one that are perpendicular to both $\mathbf{u}$ and $\mathbf{v}$. What are the two vectors?

6a. [15 pts.] In which direction should you travel for the greatest increase in the output of

$$w(x, y, z) = xy + 3yz^2$$

if you start at $(-1, -1, 9)$?

6b. [15 pts.] If y is a function of x determined by the equation $\cos(x - y) = xe^y$, what is $\frac{dy}{dx}$?

INITIALS.....

7a. [10 pts.] If f is the function

$$f(x, y, z) = \frac{1}{1 + \sqrt{x + y + z}}$$

What is the domain of f? Is it open or closed? Bounded or unbounded?

7b. [10 pts.] If h is the function

$$h(x, y, z) = \frac{1}{\sqrt{x + y + z}}$$

What is the domain of h? Is it open or closed? Bounded or unbounded?

7c. [5 pts.] What is the range of $g(x, y) = (y - \sin x)^2$

7d. [15 pts.] Draw five level sets[1] of the function $g(x, y) = (y - \sin x)^2$

[1] You are permitted to draw each level set separately.

INITIALS.....

8. [10 pts.] Does the series

$$\sum_{n=1}^{\infty} n \sin \frac{1}{n}$$

converge or diverge? Give reason(s) for your answer. (Hint: A successful solution uses one of: the ratio test, the direct comparison test, or the Nth term test.)

9. [25 pts.] Let $z = \frac{x}{y}$, $x = se^t$, and $y = 1 + se^{-t}$. (Write out the Chain Rule formula you use and a dependency diagram.)

a. Use the Chain Rule to find $\frac{\partial z}{\partial s}$.

b. Use the Chain Rule to find $\frac{\partial z}{\partial t}$.

10. [10 pts.] Use the Chain Rule to find $\frac{dz}{dt}$ if $z = x \ln(x+2y)$, $x = \sin t$, and $y = \cos t$. Write out the Chain Rule formula you use and a dependency diagram.

INITIALS.....

11. [15 pts.] Let $f(x, y, z) = -2x + \cos(yz) + e^{xyz}$. What is the directional derivative of f at $P_0 = (1, 4, -1)$ when traveling from P_0 towards $(3, 3, -1)$?

12. [8 pts.] Describe (in words) the level surfaces of the function $w(x, y, z) = x + 3y + 5z$

13. [10 pts.] Does the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n}$$

converge or diverge? Give reason(s) for your answer. (Hint: One successful solution uses one of: the Nth term test, the integral test, or the root test.)

INITIALS.....

14a. [10 pts.] What are the parametric equations of the line segment joining $(2, 0, 8)$ and $(-1, 3, 0)$?

14b. [5 pts.] What is the length of the line segment joining $(1, 1, 3)$ and $(4, 0, -5)$?

15. [20 pts.] Find the local minimum and maximum values and saddle points of

$$f(x, y) = (2x - x^2)(2y - y^2)$$

INITIALS.....

16. [20 pts.] Find the maximum and minimum values of $f(x, y, z) = xyz$, subject to the constraint $x^2 + 2y^2 + 3z^2 = 6$

17. [20 pts.] Find the local maximum and minimum values and saddle points of the function $f(x, y) = 9 - 2x + 4y - x^2 - 4y^2$. What is the global maximum and global minimum?