Name: Key

Math 21C - Quiz 1
Thursday, June 29, 2006

Question 1. Let $\{a_n\}$ be the sequence where the terms are defined:

$$a_n = \frac{\sin n}{n}$$

Does this sequence converge or diverge? Why does it converge or diverge? If it converges, what does it converge to?

$-1 \le \sin n \le 1$

$-\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$

$b_n \le a_n \le c_n$, where sequences $b_n = -\frac{1}{n} \to 0$ and $c_n = \frac{1}{n} \to 0$

By Sandwich Method/Theorem, $a_n \to 0$ also.

Question 2. Let $\{c_n\}$ be the sequence with terms defined as follows:

$$c_n = \left\{ \frac{n^4}{\ln(n) + 2006} \right\}$$

Does this sequence converge or diverge? Why does it converge or diverge? If it converges, what does it converge to?

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} \frac{n^4}{\ln n + 2006} = \lim_{n\to\infty} \frac{4n^3}{\frac{1}{n}} = \lim_{n\to\infty} 4n^4 = \infty$$

$\frac{\infty}{\infty}$, use l'Hopital

$\therefore$ So sequence c_n diverges

Question 3. Does the series

$$\sum_{n=2}^{\infty} (3 + \sin n)$$

converge or diverge? Why does it converge or diverge? If it converges, what is the sum?

The n^{th} term of the series is $a_n = 3 + \sin n$

this sequence oscillates, taking values between 2 and 4.

(not necessary to say this part)

Since a_n oscillates, it doesn't converge.

The N^{th} term test says $\sum$ diverges. (b/c $\lim_{n\to\infty} a_n \neq 0$)

#3 Alternate solution

$$-1 \leq \sin(n) \leq 1$$

add 3: $\quad 2 \leq 3 + \sin(n) \leq 4$

Since $2 \leq 3 + \sin n$

$$\sum_{n=2}^{\infty} 2 \leq \sum_{n=2}^{\infty} 3 + \sin n$$

this series clearly diverges, so by Comparison, so does this