Series (Summary)

Suppose $\sum a_n$ is your series. The **terms of the series** are $a_1, a_2, \ldots$. Associated to the series is the **sequence of partial sums**, denoted S_k, where $S_k = \sum_{something}^{k} a_n$ The series converges if S_k converges, and in this case, the entire **sum of the series** (the grand total of adding up the a_n's) is considered to be the limit of the S_k. Otherwise, the series **diverges**.

Nth term test. This should almost always be tried first. Look at the sequnce a_n (not the sequence S_k). If a_n is a seqence that either (1) does NOT coverge at all or (2) converges, but not to zero, then the series $\sum a_n$ diverges. If $a_n \to 0$, then you can't conclude anything. The Nth term test can NOT be used to conclude that a series converges.

Examining the sequence of partial sums. Sometimes, you can directly see what the S_k converge to. If so, that is the value of the sum. This could be because the series is **telescoping** (a negative part of one term of the series cancels with the positive part of another term). Series that don't look telescoping may look as such after **partial fraction decomposition**.

Geometric Series. If $\sum a_n$ is geometric with common ratio r, and $|r| < 1$, then the series converges. The sum of the series is easily calculated by this formuala. Let c represent the first term (usually a_0 or a_1, but not always). Then the sum is $\Sigma = \frac{c}{1-r}$. In the case that $|r| \geq 1$, the series diverges (in this case, the Nth term test would tell you this).

Integral Test. If $a_n = f(n)$ for some nonincreasing function f for all $n \geq N$, then

$$\sum_{n=b}^{\infty} a_n$$

and

$$\int_b^{\infty} f(x)dx$$

either both converge or both diverge.

Comparison Test. If $\sum a_n$ is a positive series ($a_n \geq 0$), then we can try to compare it to other series. If $\sum b_n$ diverges and $a_n \geq b_n$ for $n \geq N$, then $\sum a_n$ diverges as well. If $\sum c_n$ converges and $a_n \leq c_n$ for $n \geq N$, then $\sum a_n$ converges as well.

Ratio Test. If $\sum a_n$ is a strictly positive series ($a_n > 0$), let

$$\rho = \lim_{n\to\infty} r_n = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

If $\rho < 1$, then $\sum a_n$ converges. If $\rho > 1$, $\sum a_n$ diverges. If $\rho = 1$, the test is inconclusive.

Root Test. If $\sum a_n$ is a positive series ($a_n \geq 0$), let

$$\rho = \lim_{n\to\infty} r_n = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$$

If $\rho < 1$, then $\sum a_n$ converges. If $\rho > 1$, $\sum a_n$ diverges. If $\rho = 1$, the test is inconclusive.

Alternating Series Test. If $\sum a_n$ is an alternating series (every other term switches between positive and negative), set $p_n = |a_n|$. If $p_n \geq p_{n+1}$ for all $n \geq N$ and

$$p_n \to 0$$

then $\sum a_n$ converges. The alternativg series test can not be used to conclude that an alternating series (or any series, for that matter) diverges.

Absolutely Convergent Series Theorem. If $\sum a_n$ is a series, set $p_n = |a_n|$. If $\sum p_n$ converges, then $\sum a_n$ converges too (and we use the terminology to say that $\sum a_n$ **converges absolutely**). If, on the other hand $\sum p_n$ diverges, the test does not conclude anything.