

# Cameos of Multivariable Calculus and Series

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# Preface

## In General

This workbook offers a variety of opportunities for students to think in different ways about material related to various aspects of Calculus. Some of the topics addressed supplement what is offered in the text; this includes discussion of material prerequisite to the study of Calculus and treatment of the process of mathematics in general. Most of the material in Chapters 3 and 4 complements the text; the discussion sets and project seek to enhance reflection on the material covered. Rather than being in an examples—theorem—proof—examples format that is to be read, this volume is in a format that seeks to engage you in exploration; you are asked to think about the development of ideas encountered and to write about what is being offered for study. Concurrent experience with both formats should allow you to “dig deeper and reach higher” with all your mathematical experiences.

## About the Discussion Sets and Project

The discussion sets and project contained in this volume had their beginnings in the UC-Davis Calculus Revitalization Project (CaRP) that was conceived in the Spring quarter of 1993. After a call to our faculty and graduate students for comments, suggestions and descriptions of successful calculus learning experiences, a group of mathematics faculty decided to focus its first efforts on improving mathematics and calculus literacy. This was seen as a first and important step towards enabling our students to make meaningful use of technology.

In the summer of 1993, three of the contributors co-authored a set of labs that were used during the 93–94 academic year. A fourth contributor did most of the background work that went into the historical snapshots or Mathstories. The project in Chapter 4 was co-authored by the editors who were co-directors for the Revitalization effort. The focus of the labs and the project was to use work on selected topics to improve the students’ ability to read, write, and talk math. The project offers an opportunity for an intense, extended experience with the mathematical development of a topic.

After the labs and project were used by 500–750 students, comments from students,

teaching assistants, and course instructors on the labs provided valuable feedback. In the summer of 1994, five of the contributors formed a CaRP writing team. The team accomplished a major revision of the labs including replacement of some units and minor reformatting of others. The resulting Discussion Sets were used during the 1994–95 academic year. Throughout the Revitalization Project, the editors served as architects for the basic design of the materials and as contributing authors.

Comments from students, teaching assistants, and instructors prompted further minor revisions. The editors made those revisions as well as other ones that were needed for text independence. The versions of the Discussion Sets and Project that are in this volume reflect revisions that were based on at least four uses of the materials.

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