

Math 67
Homework 1 Due Friday January 12th

IMPORTANT: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS exercises don't have to be hand-in. I separated the list of exercises in the three categories for your convenience.

1. Solve all calculational exercises in chapter 1 (page 9)
2. Solve calculational exercises 1,2,4,5(a,d), 6(a,d), 7. from chapter 2

3. Solve all Proof-writing exercises in chapter 1 (page 10)
4. Solve proof-writing exercises 2, 3, and 5 from chapter 2.
5. In the planet ZZ the population has a different way to add two real numbers where $a\overline{+}b = 2a + b$ (the $\overline{+}$ sign is on the right-hand side symbol is the earthlings way to do addition!!) Is this operation commutative or associative? why or why not?

6. (BONUS) We defined in class addition and multiplication "modulo 6". Recall that this is done with the numbers 0, 1, 2, 3, 4, 5, 6 you add and multiply the usual way but then the final answer is the remainder of dividing by 6. For example $4 \cdot 5$ is normally equal 20 but modulo 6 (divide by 6) the answer has remainder 2, thus $4 \cdot 5 = 2$. Similarly $2 \cdot 3 = 0$ and $4 + 4 = 2$, $3 + 4 = 1$, etc. Do the numbers 0, 1, 2, 4, 5, 6 with these operations form a field? How about modulo 7 addition and multiplication with the numbers 0, 1, 2, 3, 4, 5, 6, 7? Check all the properties of a field!!!
7. (BONUS): Can you invent a field with only 4 elements? Can you invent a field with 6 elements? why?