Math 67
Homework 3 Due January 30th

IMPORTANT 1: The First midterm exam is next friday January 30th. Some tips:

1) Make sure you read and understand Chapters 1-4 and 12.1-12.3 in the book. These are the topics we will cover in the exam.

2) Learn how to use the key theorems in those chapters.

3) There will be 5 questions in the exam.

IMPORTANT 2: Please hand-in calculational exercises separated from proof-writing exercises (there will be two piles). BONUS/PRACTICE exercises (if any) don't have to be handed-in. I separated the list of exercises in the three categories for your convenience.

——————CALCULATION——————

1. Solve all calculational exercises 1,3,4 in Chapter 4.

2. for each of the following matrices decide whether is invertible or not and find the inverse when possible

$$A_1 \begin{bmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{bmatrix} \qquad A_2 \begin{bmatrix} 6 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{bmatrix}$$

3. Determine whether the vector $(3, -1, 0, -1)$ belongs to the subspace generated by all vectors $(2, -1, 3, 2)$, $(-1, 1, 1, -3)$, $(1, 1, 9, -5)$?

4. Consider the vector subspaces $U_1 = \{x | A_1 x = 0\}$ and $U_2 = \{x | A_2 x = 0\}$ The matrices A_1, A_2 are those of exercise 2 above. Describe all the vectors that belong to the intersection subspace $U_1 \cap U_2$.

——————PROOF-WRITING——————

5. Solve all proof-writing exercises in Chapter 4 (1-4).

6. Let V the vector space of all polynomials $R[x]$ which of the following are subspaces:

   a) Set of all $f \in R[x]$ with $f(0) = f(1)$ b) Set of all $f \in R[x]$ with $f(3) = 1 + f(-5)$ c) Set of all $f \in R[x]$ with $f(x^2) = f(x)^2$ d) Set of all $f \in R[x]$ with $f(-1) = 0$.

7. Let V be the vector space of all $n \times n$ matrices. Which of the following subsets of V are actually vector subspaces?

   a) All invertible matrices b) All non-invertible matrices c) All those for which $A^2 = A$.

8. In R^3 consider the subspace $W_1 = \{(x, y, z) \in R^3 | x = y = 0\}$Encuentre W_2 subspace of R^3 such that $W_1 + W_2 = R^3$. Can you make that a direct sum?

9. (PRACTICE) In the following statements state whether it is true or false and give a short reason:

   a) There are systems of linear equations with 7 solutions.

   b) There are polynomials of arbitrarily large degree with a single root at i.

   c) A 3×7 matrix A has a only a trivial solution for $Ax = 0$.

   d) The modulus of a complex number is the same as the modulus of its conjugate

   e) The conjugate of a product is the product of the conjugates.

10. (PRACTICE) Find the cubic root of the number $4\sqrt{2} + 4\sqrt{2}i$

11. (PRACTICE) This problem has three closely related parts.

    a) Find all the solutions to the system of equations $Ax = b$, where

    $$A := \begin{bmatrix} 1 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & -1 \end{bmatrix} \text{ and } b := \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

    b) Is the matrix A invertible?

    c) Suppose now that b is the zero vector. What are the solutions now?

12. (PRACTICE) Your uncle tells you that the 3×3 matrix A is invertible, he shows you its inverse A^{-1} but he lost A! Now another matrix B was obtained from A by substracting 4 times row 1 of A from row 3. Find B^{-1}.

13. (PRACTICE) Suppose U_1, U_2 are subspaces of a vector space V. Under which conditions is it true that U_1 union U_2 is a subspace?