HW6, due May 27

(1) Two stocks are available. The corresponding expected rates of return are $\bar{r}_1, \bar{r}_2$; the corresponding variances and covariances are $\sigma_1^2, \sigma_2^2, \sigma_{12}$. What percentages of total investment should be invested in each of the two stocks to minimize the total variance of the rate of return of the resulting portfolio? What is the mean rate of return of this portfolio?

(2) There are just three assets with rates of return r_1, r_2, r_3, respectively. The covariance matrix and the expected rate of return are

$$\mathbf{C} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \bar{\mathbf{r}} = \begin{bmatrix} 0.4 \\ 0.8 \\ 0.8 \end{bmatrix}$$

(a) Find the minimum variance portfolio (MVP). [Hint: by symmetry $w_1 = w_3$]
(b) Find another efficient portfolio by setting $\lambda = 1, \mu = 0$. What is the expected return of this portfolio? (Make sure the weights sum to one by normalizing).
(c) If the risk-free rate $r_f = 0.2$, find the efficient portfoilio of risky assets.

(3) Suppose there are n assets which are uncorrelated. You may invest in any one or in any combination of them. The mean rate of return $\bar{r}$ is the same for each asset, but the variances are different. The return on asset i has a variance of $\sigma_i^2, i = 1, ..., n$.
(a) Describe the feasible set and the efficient frontier on the $\bar{r} - \sigma$ plane.
(b) Find the minimum-variance point. Express your result in terms of

$$\bar{\sigma}^2 = \left(\sum_{i=1}^{n} \sigma_i^{-2} \right)^{-1}.$$

(c) Suppose the total amount of asset i in the market is X_i. Let $T = \sum_{i=1}^{n} X_i$ and set $x_i = X_i/T$. x_i is the market share of asset i. We can think of the **market portfolio** in the normalized form as $\mathbf{x} = (x_1, x_2, ..., x_n)$. Let r_f be the risk-free rate. Find an expression for β_i in terms of the x_i's and σ_i's.

(4) Assume that the expected rate of return on the market portfolio is 23% and the rate of return on T-bills (the risk-free rate) is 7%. The standard deviation of the market is 32%. Assume that the market portfolio is efficient.
(a) What is the equation of the market line?
(b) If an expected return of 39% is desired, what is the standard deviation of this position?
(c) If you have \$1000 to invest, how should you allocate it to achieve the above position?
(d) If you invest \$300 in the risk-free asset and \$700 in the market portfolio, how much money should you expect to have at the end of the year?

(5) Consider a world with only two risky assets and a risk-free asset. The two risky assets are in equal supply (i.e. the same number of outstanding

shares) in the market. The following information is known: $r_f = 0.1, \sigma_1^2 = 0.04, \sigma_{12} = 0.01, \sigma_2^2 = 0.02, \bar{r}_M = 0.18$.
(a) Find a general expression for $\sigma_M^2, \beta_1, \beta_2$.
(b) According to CAPM, what are the numerical values of $\bar{r}_1, \bar{r}_2$?