

HW1, Part I (due April 8)

- (1) Consider an investment opportunity that generates the following stream of cash flow

$$-1000, -1200, 800, 900, 800.$$

Suppose you can **borrow and/or lend** money at the interest rate 6% at will. Will you go for the above investment opportunity?

- (2) Let P_j be the principal reduction of a mortgage (loan amount L with monthly interest rate r) in the j -th payment. Show that

$$P_j = \frac{Lr(1+r)^{j-1}}{(1+r)^n - 1}$$
$$L = \sum_{j=1}^n P_j$$

where n is the total number of payments.

- (3) Let (x_1, x_2, \dots, x_n) and (y_1, y_2, \dots, y_n) be two streams of cash flow. Let

$$X_j = \sum_{i=1}^j x_i, \quad \bar{X}_j = \sum_{i=1}^j X_j,$$
$$Y_j = \sum_{i=1}^j y_i, \quad \bar{Y}_j = \sum_{i=1}^j Y_i,$$

for $j = 1, \dots, n$.

Suppose the interest rate r is nonnegative. Prove that if $X_n \geq Y_n$ and $\bar{X}_j \geq \bar{Y}_j, j = 1, \dots, n$, then the first stream is more profitable (i.e. has larger present value).