HW 9

8.2 (2) $\int \theta \cos \pi\theta \; d\theta$

Let $u = \theta \Rightarrow du = 1 \; d\theta$

$dv = \cos \pi\theta \Rightarrow v = \frac{1}{\pi} \sin \pi\theta$

$\int \theta \cos \pi\theta \, d\theta = \frac{\theta}{\pi} \sin \pi\theta - \int \frac{1}{\pi} \sin \pi\theta \, d\theta$

$= \frac{\theta}{\pi} \sin \pi\theta - \frac{1}{\pi^2}(-\cos \pi\theta)$

$= \boxed{\frac{\theta}{\pi} \sin \pi\theta + \frac{1}{\pi^2} \cos \pi\theta + C}$

(6) $\int_1^e x^3 \ln x \; dx$

Let $u = \ln x \Rightarrow du = \frac{1}{x} \, dx$

$dv = x^3 \Rightarrow v = \frac{1}{4} x^4$

$\int_1^e x^3 \ln x \, dx = \frac{1}{4} x^4 \ln x \Big|_1^e - \int_1^e \frac{1}{4} x^3 \, dx$

$= \frac{1}{4} e^4 \cdot 1 - \frac{1}{4} \cdot 0 - \left[\frac{1}{16} x^4 \Big|_1^e \right]$

$= \frac{1}{4} e^4 - \left(\frac{1}{16} e^4 - \frac{1}{16}\right) = \boxed{\frac{3}{16} e^4 + \frac{1}{16}}$

(8) $\int \sin^{-1} y \; dy$

Let $u = \sin^{-1} y \Rightarrow du = \frac{1}{\sqrt{1-y^2}} \, dy$

$dv = 1 \, dy \Rightarrow v = y$

$\int \sin^{-1} y \, dy = y \sin^{-1} y - \int \frac{y}{\sqrt{1-y^2}} \, dy$

Subst: $u = 1 - y^2 \Rightarrow du = -2y \, dy$

So we have $y \sin^{-1} y + \frac{1}{2} \int u^{-1/2} \, du$

$= y \sin^{-1} y + \frac{1}{2} \cdot 2 u^{1/2} + C$

$= \boxed{y \sin^{-1} y + \sqrt{1-y^2} + C}$

(16) $\int t^2 e^{4t} \, dt = \boxed{\frac{1}{4} t^2 e^{4t} - \frac{1}{8} t e^{4t} + \frac{1}{32} e^{4t} + C}$

f and derivs		g and int's
t^2	+	e^{4t}
$2t$	−	$\frac{1}{4} e^{4t}$
2	+	$\frac{1}{16} e^{4t}$
0	−	$\frac{1}{64} e^{4t}$

(22) $\int e^{-y} \cos y \, dy$

Let $u = e^{-y} \Rightarrow du = -e^{-y} dy$

$dv = \cos y \, dy \Rightarrow v = \sin y$

$e^{-y} \sin y - \int -e^{-y} \sin y \, dy = \int e^{-y} \cos y \, dy$

$u = -e^{-y} \Rightarrow du = e^{-y} dy$

$dv = \sin y \, dy \Rightarrow v = -\cos y$

$\int e^{-y} \cos y \, dy = e^{-y} \sin y - \left[e^{-y} \cos y + \int e^{-y} \cos y \, dy \right]$

$= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y \, dy$

$\Rightarrow 2 \int e^{-y} \cos y \, dy = e^{-y} (\sin y - \cos y)$

$\Rightarrow \int e^{-y} \cos y \, dy = \boxed{\frac{1}{2} e^{-y} (\sin y - \cos y) + C}$

(26) $\int_0^1 x \sqrt{1-x} \, dx$

Let $u = x$, $du = dx$

$dv = \sqrt{1-x} \, dx \Rightarrow v = -\frac{2}{3} (1-x)^{3/2}$

$\int_0^1 x \sqrt{1-x} \, dx = -\frac{2}{3} x \sqrt{(1-x)^3} \Big|_0^1 + \int_0^1 \frac{2}{3} \sqrt{(1-x)^3} \, dx$

$= (0-0) + \frac{2}{3} \left(-\frac{2}{5} (1-x)^{5/2} \right) \Big|_0^1$

$= \frac{-4}{15} (0-1) = \boxed{\frac{4}{15}}$

(28) $\int \ln(x+x^2) \, dx$

$u = \ln(x+x^2) \Rightarrow du = \frac{1+2x}{x+x^2} dx$

$dv = 1 \, dx \Rightarrow v = x$

$\int \ln(x+x^2) \, dx = x \cdot \ln(x+x^2) - \int \frac{x+2x^2}{x+x^2} dx$

$= x \cdot \ln(x+x^2) - \int \frac{1+2x}{1+x} dx$

$= x \cdot \ln(x+x^2) - \int \frac{2(x+1)-1}{1+x} dx$

$= x \cdot \ln(x+x^2) - (2x - \ln(1+x))$

$= \boxed{x \cdot \ln(x+x^2) - 2x + \ln(1+x) + C}$

$\boxed{8.2}$ 30 $\int z(\ln z)^2\,dz$

$u = (\ln z)^2 \Rightarrow du = \frac{2\ln z}{z}\,dz$

$dv = z\,dz \Rightarrow v = \frac{1}{2}z^2$

$\int z(\ln z)^2 dz = \frac{1}{2}z^2(\ln z)^2 - \int z\cdot\ln z\,dz$

$u = \ln z \Rightarrow du = \frac{1}{z}\,dz$

$dv = z\,dz \Rightarrow v = \frac{1}{2}z^2$

$\int z(\ln z)^2 dz = \frac{1}{2}z^2(\ln z)^2 - \left[\frac{1}{2}z^2\ln z - \int \frac{1}{2}z\,dz\right]$

$= \frac{1}{2}z^2(\ln z)^2 - \frac{1}{2}z^2\ln z - \frac{1}{4}z^2 + C$

$= \boxed{\frac{1}{2}z^2\left[(\ln z)^2 - \ln z - \frac{1}{2}\right] + C}$

36 $y = x\sin x, \quad 0 \le x \le \pi$

a) about the y-axis: $(x=0)$

$V = 2\pi\int_0^\pi x\cdot x\sin x\,dx$

x^2	+	$\sin x$
$2x$	−	$-\cos x$
2	+	$-\sin x$
0	−	$\cos x$

$V = 2\pi\left[-x^2\cos x + 2x\sin x + 2\cos x\Big|_0^\pi\right]$

$= 2\pi\left[-\pi^2(-1) + 0 - 2 - (2)\right]$

$= \boxed{2\pi(\pi^2 - 4)}$

b) about the line $x = \pi$:

$V = 2\pi\int_0^\pi (\pi - x)\,x\sin x\,dx$

$= 2\pi\left[\int_0^\pi \pi x\sin x\,dx - \int_0^\pi x^2\sin x\,dx\right]$

$= 2\pi^2\int_0^\pi x\sin x\,dx$ − (answer from above)

x	+	$\sin x$
1	−	$-\cos x$
0	+	$-\sin x$

$= 2\pi^2\left(-x\cos x + \sin x\Big|_0^\pi\right) - 2\pi(\pi^2 - 4)$

$= 2\pi^2(\pi) - 2\pi(\pi^2 - 4)$

$= 2\pi^3 - 2\pi^3 + 8\pi = \boxed{8\pi}$

(38) $y = 4e^{-t}(\sin t - \cos t), \quad t \geq 0$

$$\text{Avg value} = \frac{1}{2\pi - 0}\int_0^{2\pi} 4e^{-t}(\sin t - \cos t)\,dt$$

$$= \underbrace{\frac{1}{2\pi}\int_0^{2\pi} 4e^{-t}\sin t\,dt}_{(\mathrm{I})} - \underbrace{\frac{1}{2\pi}\int_0^{2\pi} 4e^{-t}\cos t\,dt}_{(\mathrm{II})}$$

(I): $\int_0^{2\pi} e^{-t}\sin t\,dt = -e^{-t}\sin t\Big|_0^{2\pi} - \int -e^{-t}\cos t\,dt$

$\left(\begin{array}{l} u = \sin t \Rightarrow du = \cos t\,dt \\ dv = e^{-t}dt \Rightarrow v = -e^{-t} \end{array}\right)$

This will cancel w/integral in (II)

$$\text{Avg Value} = \frac{4}{2\pi}\left(e^{-t}\sin t\Big|_0^{2\pi} + \int e^{-t}\cos t\,dt\right) - \frac{4}{2\pi}\int e^{-t}\cos t\,dt$$

$$= \frac{4}{2\pi}\left(e^{-2\pi}\cdot 0 - 1\cdot 0\right)$$

$\boxed{\text{Avg. Value} = 0}$

$\boxed{\S 8.3}$

(12) $\dfrac{2x+1}{x^3-7x+12} = \dfrac{A}{x-4} + \dfrac{B}{x-3} \Rightarrow 2x+1 = A(x-3) + B(x-4)$

$\Rightarrow (A+B)x - 3A - 4B = 2x+1$

$\Rightarrow A + B = 2, \quad -3A - 4B = 1$

$A = 2 - B \Rightarrow -3(2-B) - 4B = -6 + 3B - 4B = -6 - B = 1$

$\Rightarrow B = -7 \Rightarrow A = 9.$

So $\displaystyle\int \frac{2x+1}{x^3-7x+12}\,dx = \int \frac{9}{x-4} - \frac{7}{x-3}\,dx$

$= 9\ln|x-4| - 7\ln|x-3| + C$

$= \boxed{\ln\left|\dfrac{(x-4)^9}{(x-3)^7}\right| + C}$

(16) $\dfrac{x+3}{2x^3-8x} = \left(\dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}\right) \Rightarrow \frac{1}{2}(x+3) = A(x+2)(x-2) + Bx(x-2) + Cx(x+2)$

If $x = 0$, then $\dfrac{3}{2} = -4A \Rightarrow A = -\dfrac{3}{8}$

$x = -2$, then $8B = \frac{1}{2} \Rightarrow B = \frac{1}{16}$

$x = 2$ then $8C = \frac{5}{2} \Rightarrow C = \frac{5}{16}$,

So $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx = \int -\frac{3}{8}\cdot\frac{1}{x}\,dx + \frac{1}{16}\int\frac{1}{x+2}\,dx + \frac{5}{16}\int\frac{1}{x-2}\,dx$

$= -\frac{3}{8}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{5}{16}\ln|x-2| + C$

$= \boxed{\dfrac{1}{16}\ln\left|\dfrac{(x+2)(x-2)^5}{x^6}\right| + C}$

(18) $\dfrac{x^3}{x^2-2x+1} = x+2 + \dfrac{3x-2}{x^2-2x+1}$ (after long division).

$\dfrac{3x-2}{x^2-2x+1} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} \Rightarrow Ax - A + B = 3x-2$

$\Rightarrow A = 3, \quad -A+B = -2 \Rightarrow -3+B = -2 \Rightarrow B = 1.$

So $\displaystyle\int_{-1}^{0} \frac{x^3}{x^2-2x+1}\,dx = \int_0^1 x+2 + \frac{3}{x-1} + \frac{1}{(x-1)^2}\,dx$

$= \frac{1}{2}x^2 + 2x + 3\ln|x-1| - \dfrac{1}{x-1} \Big|_{-1}^{0}$

$= 3\ln 1 + 1 - \left(\frac{1}{2} - 2 + 3\ln 2 + \frac{1}{2}\right)$

$= \boxed{2 - 3\ln 2}$

(20) $\dfrac{x^2}{(x-1)(x^2+2x+1)} = \dfrac{A}{x-1} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$

$\Rightarrow x^2 = A(x^2+2x+1) + B(x^2-1) + Cx - C$

$x = -1 \Rightarrow -2C = 1 \Rightarrow C = -\frac{1}{2}$

$x = 1 \Rightarrow 4A = 1 \Rightarrow A = \frac{1}{4}$

$(A+B)x^2 = x^2 \Rightarrow A+B = 1 \Rightarrow B = \frac{3}{4}.$

So $\displaystyle\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx = \int \frac{1}{4}\cdot\frac{1}{x-1} + \frac{3}{4}\cdot\frac{1}{x+1} - \frac{1}{2}\cdot\frac{1}{(x+1)^2}\,dx$

$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2}\cdot\frac{1}{x+1} + C$

$= \boxed{\frac{1}{4}\ln\left|(x-1)(x+1)^3\right| + \frac{1}{2(x+1)} + C}$

(24) $\dfrac{8x^2+8x+2}{(4x^2+1)^2} = \dfrac{Ax+B}{4x^2+1} + \dfrac{Cx+D}{(4x^2+1)^2}$

$\Rightarrow (Ax+B)(4x^2+1) + Cx + D = 8x^2+8x+2$

$\Rightarrow 4Ax^3 + 4Bx^2 + Ax + B + Cx + D = 8x^2+8x+2$

$4Ax^3 = 0x^3 \Rightarrow A = 0 \qquad (A+C)x = 8x \Rightarrow C = 8$

$4Bx^2 = 8x^2 \Rightarrow B = 2 \qquad B + D = 2 \Rightarrow D = 0.$

So $\displaystyle\int \frac{2}{4x^2+1} + \frac{8x}{(4x^2+1)^2}\,dx = \boxed{\tan^{-1}(2x) - \frac{1}{4x^2+1} + C}$

(28) $$\frac{\theta^4-4\theta^3+2\theta^2-3\theta+1}{(\theta^2+1)^3} = \frac{A\theta+B}{\theta^2+1} + \frac{C\theta+D}{(\theta^2+1)^2} + \frac{E\theta+F}{(\theta^2+1)^3}$$

$$\Rightarrow \theta^4-4\theta^3+2\theta^2-3\theta+1 = (A\theta+B)(\theta^2+1)^2 + (C\theta+D)(\theta^2+1) + E\theta+F$$

$$\Rightarrow (A\theta+B)(\theta^4+2\theta^2+1) + C\theta^3+D\theta^2+C\theta+D+E\theta+F$$

$$\Rightarrow A\theta^5 + B\theta^4 + 2A\theta^3 + 2B\theta^2 + A\theta + B + C\theta^3 + D\theta^2 + C\theta + D + E\theta + F$$

$$= \theta^4-4\theta^3+2\theta^2-3\theta+1$$

$$\Rightarrow A=0;\quad B=1;\quad C=-4;\quad 2B+D=2 \Rightarrow D=0;$$

$$A+C+E=-3 \Rightarrow E=1;\quad B+D+F=1 \Rightarrow F=0.$$

So $$\int \frac{1}{\theta^2+1} - \frac{4\theta}{(\theta^2+1)^2} + \frac{\theta}{(\theta^2+1)^3}\,d\theta$$

$$= \boxed{\tan^{-1}\theta + \frac{2}{\theta^2+1} - \frac{1}{4}\cdot\frac{1}{(\theta^2+1)^2} + C}$$

(30) $$\frac{x^4}{x^2-1} = x^2+1+\frac{1}{x^2-1} = x^2+1+\frac{A}{x-1}+\frac{B}{x+1}$$

$$\Rightarrow Ax+A+Bx-B=1$$

$$\Rightarrow A+B=0,\quad A-B=1$$

$$A=\frac{1}{2},\quad B=-\frac{1}{2}$$

$$\int x^2+1+\frac{1}{2(x-1)}-\frac{1}{2(x+1)}\,dx = \frac{1}{3}x^3+x+\frac{1}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+C$$

$$= \boxed{\frac{1}{3}x^3+x+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C}$$

(34) $$\frac{2y^4}{y^3-y^2+y-1} = 2y+2+\frac{2}{y^3-y^2+y-1} = 2y+2+\frac{A}{y-1}+\frac{By+C}{y^2+1}$$

$$\Rightarrow Ay^2+A+(By+C)(y-1) = Ay^2+A+By^2-By+Cy-C = 2$$

$$\Rightarrow A+B=0,\quad -B+C=0,\quad A-C=2$$

$$\Rightarrow B=-A,\quad -(-A)+C=0 \Rightarrow A+C=0 \Rightarrow 2A=2 \Rightarrow A=1,$$

$$B=-1,\quad C=-1$$

$$\int 2y+2+\frac{1}{y-1}-\frac{y}{y^2+1}-\frac{1}{y^2+1}\,dy$$

$$= \boxed{y^2+2y+\ln|y-1|-\frac{1}{2}\ln(y^2+1)-\tan^{-1}y+C}$$