(52) $\int \frac{P(x)}{x^3(x-1)^2}\,dx$

$P(x) = ax^2+bx+C$. $P(0) = 1 \Rightarrow C = 1$.

$P'(0) = 2a(0)+b = 0 \Rightarrow b = 0$, so

$P(x) = ax^2+1$.

So $\int \frac{ax^2+1}{x^3(x-1)^2}\,dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{E}{(x-1)^2}\,dx$.

For this to be rational, $A=0$ and $D=0$.

So $\int \frac{ax^2+1}{x^3(x-1)^2}\,dx = \int \frac{B}{x^2} + \frac{C}{x^3} + \frac{E}{(x-1)^2}\,dx$

$\Rightarrow ax^2+1 = Bx(x^2-2x+1) + C(x^2-2x+1) + Ex^3$

$\Rightarrow ax^2+1 = Bx^3 - 2Bx^2 + Bx + Cx^2 - 2Cx + C + Ex^3$

x^3: $B+E=0$ $\qquad$ x: $B-2C=0$

x^2: $-2B+C=a$ $\qquad$ 1: $\boxed{C=1}$

$\Rightarrow B-2=0 \Rightarrow \boxed{B=2}$

$\Rightarrow -2(2)+1=a \Rightarrow \boxed{a=-3}$

and $\boxed{E=-2}$

So $\boxed{P(x) = -3x^2+1}$

§8.4

(2) $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx = \int_0^{\pi} \sin\left(\frac{x}{2}\right)dx - \int_0^{\pi} 2\cos^2\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx + \int_0^{\pi}\cos^4\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)dx$

$= -2\cos\left(\frac{x}{2}\right) + \frac{4}{3}\cos^3\left(\frac{x}{2}\right) - \frac{2}{5}\cos^5\left(\frac{x}{2}\right)\Big|_0^{\pi}$

$= -\left(-2+\frac{4}{3}-\frac{2}{5}\right) = \frac{30}{15} - \frac{20}{15} + \frac{6}{15} = \boxed{1\frac{1}{15}}$

(12) $\int_0^{\pi} \cos^2 2x \sin 2x\,dx$ $\qquad$ Let $u = \cos 2x \Rightarrow du = -2\sin 2x\,dx$.

So $-\frac{1}{2}\int_0^{\pi} u^2\,du = -\frac{1}{6}u^3\,du = -\frac{1}{6}\cos^3 2x\Big|_0^{\pi}$

$= -\frac{1}{6}+\frac{1}{6} = \boxed{0}$

(14) $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta \, d\theta = \int_0^{\pi/2} \sin^2 2\theta (1-\sin^2 2\theta)\cos 2\theta \, d\theta$

$= \int_0^{\pi/2} \sin^2 2\theta \cos 2\theta - \int_0^{\pi/2} \sin^4 2\theta \cos 2\theta \, d\theta$

$= \frac{1}{6} \sin^3 2\theta - \frac{1}{10} \sin^5 2\theta \Big|_0^{\pi/2}$

$= \boxed{0}$

(16) $\int_0^{\pi} \sqrt{1-\cos 2x} \, dx = \int_0^{\pi} \sqrt{2\sin^2 x} \, dx = \sqrt{2} \int_0^{\pi} |\sin x| \, dx$

$= \sqrt{2}(-\cos x)\Big|_0^{\pi} = \sqrt{2}(1+1) = \boxed{2\sqrt{2}}$

(22) $\int_{-\pi}^{\pi} (1-\cos^2 t)^{3/2} \, dt = \int_{-\pi}^{\pi} |\sin^3 t| \, dt = \int_{-\pi}^{0} -\sin^3 t \, dt + \int_0^{\pi} \sin^3 t \, dt$

$= \int_0^{\pi} \sin t (1-\cos^2 t) \, dt = \int_0^{\pi} \sin t \, dt - \int_0^{\pi} \sin t \cos^2 t \, dt$

$- \int_{-\pi}^{0} \sin t \, dt + \int_{-\pi}^{0} \sin t \cos^2 t$

$= \left(-\cos t + \frac{1}{3}\cos^3 t \Big|_0^{\pi}\right) + \left(\cos t - \frac{1}{3}\cos^3 t \Big|_{-\pi}^{0}\right)$

$= 1 - \frac{1}{3} + 1 - \frac{1}{3} + \left(1 - \frac{1}{3} + 1 - \frac{1}{3}\right)$

$= 2\left(\frac{4}{3}\right) = \boxed{\frac{8}{3}}$

(26) $\int_0^{\pi/12} 3 \sec^4 3x \, dx = \int_0^{\pi/12} 3(1+\tan^2 3x)\sec^2 3x \, dx$

$= \int_0^{\pi/12} 3 \sec^2 3x + \int_0^{\pi/12} 3 \tan^2 3x \sec^2 3x$

$= \tan 3x + \frac{1}{3} \tan^3 3x \Big|_0^{\pi/12}$

$= 1 + \frac{1}{3} = \boxed{4/3}$

(34) $\int_0^{\pi/2} \sin 2x \cos 3x \, dx = \int_0^{\pi/2} \frac{1}{2}\left[\sin(2-3)x + \sin(2+3)x\right] dx$

$= \frac{1}{2} \int_0^{\pi/2} \sin(-x) \, dx + \frac{1}{2} \int_0^{\pi/2} \sin 5x \, dx$

$= \frac{1}{2} \cos(-x) - \frac{1}{10} \cos 5x \Big|_0^{\pi/2}$

$= 0 - \left(\frac{1}{2} - \frac{1}{10}\right) = -\frac{1}{2} + \frac{1}{10} = \boxed{-\frac{2}{5}}$

(40) $y = \ln(\cos x), \quad 0 \le x \le \pi/3$

$$L = \int_0^{\pi/3} \sqrt{1+(y')^2} = \int_0^{\pi/3} \sqrt{1+(-\tan x)^2}\, dx$$

$$= \int_0^{\pi/3} \sqrt{1+\tan^2 x}\, dx = \int_0^{\pi/3} \sqrt{\sec^2 x}\, dx$$

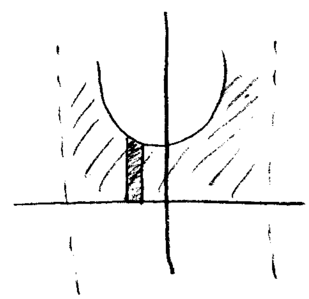
$$= \int_0^{\pi/3} |\sec x|\, dx = \int_0^{\pi/3} \sec x\, dx$$

$$= \ln\left[|\sec x + \tan x|\right]_0^{\pi/3}$$

$$= \ln\left[|\sqrt{3}+2|\right] - \ln\left[|1+0|\right]$$

$$= \boxed{\ln(2+\sqrt{3})}$$

(42) $(\tilde{x}, \tilde{y}) = \left(x, \frac{\sec x}{2}\right)$



Length $= \sec x$

Width $= dx$

So $\bar{x} = 0$ by symmetry

$$\bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} = \frac{\int_{-\pi/4}^{\pi/4} \frac{1}{2}\sec^2 x\, dx}{\int_{-\pi/4}^{\pi/4} \sec x\, dx}$$

$$= \frac{\frac{1}{2}\tan x \Big|_{-\pi/4}^{\pi/4}}{\ln|\sec x + \tan x| \Big|_{-\pi/4}^{\pi/4}} = \frac{\frac{1}{2}(1+1)}{\ln(\sqrt{2}+1) - \ln(\sqrt{2}-1)} = \frac{1}{\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)}$$

$$= \left[\ln \frac{\sqrt{2}+1}{\sqrt{2}-1}\right]^{-1}$$

$$\boxed{\text{So } (\bar{x}, \bar{y}) = \left(0, \left[\ln \frac{\sqrt{2}+1}{\sqrt{2}-1}\right]^{-1}\right)}$$

(44) $$\int_0^{\pi} \sqrt{1+\cos 4x}\, dx = \int_0^{\pi} \sqrt{2\cos^2 2x} = \sqrt{2}\int_0^{\pi} |\cos 2x|\, dx$$

$$= \sqrt{2}\left(\tfrac{1}{2}\sin 2x \Big|_0^{\pi/4}\right) - \sqrt{2}\left(\tfrac{1}{2}\sin 2x \Big|_{\pi/4}^{3\pi/4}\right) + \sqrt{2}\left(\tfrac{1}{2}\sin 2x \Big|_{3\pi/4}^{\pi}\right)$$

$$= \sqrt{2}\left(\tfrac{1}{2} - 0\right) - \sqrt{2}\left(-\tfrac{1}{2} - \tfrac{1}{2}\right) + \sqrt{2}\left(0 + \tfrac{1}{2}\right)$$

$$= \frac{\sqrt{2}}{2} + \sqrt{2} + \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2}}$$

$\boxed{\text{HW 10}}$

$\boxed{\S 8.5}$

(2) $\int \frac{3\,dy}{\sqrt{1+9y^2}}$ Let $x = 3y \Rightarrow dx = 3\,dy$

$\int \frac{dx}{\sqrt{1+x^2}}$ $x = \tan t \Rightarrow dx = \sec^2 t = \frac{1}{\cos^2 t}\,dt$

$\int \frac{1}{\cos^2 t \sqrt{1+\tan^2 t}}\,dt = \int \frac{1}{\cos^2 t \cdot \sec t} = \int \frac{1}{\cos t}\,dt$

$= \int \sec t\,dt = \ln|\sec t + \tan t| + C$

$= \ln\left|\sqrt{1+x^2} + x\right| + C = \boxed{\ln\left|\sqrt{1+9y^2} + 3y\right| + C}$

(8) $t = \frac{1}{3}\sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2};\ dt = \frac{1}{3}\cos\theta\,d\theta,$

$\sqrt{1-9t^2} = \cos\theta$

$\int \sqrt{1-9t^2}\,dt = \frac{1}{3}\int \cos^2\theta\,d\theta = \frac{1}{6}(\theta + \sin\theta\cos\theta) + C$

$= \boxed{\frac{1}{6}\left[\sin^{-1}(3t) + 3t\sqrt{1-9t^2}\right] + C}$

(14) $x = \sec\theta,\ 0 < x < \pi/2,\ dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$\int \frac{2\,dx}{x^3\sqrt{x^2-1}} = \int \frac{2\sec\theta\tan\theta\,d\theta}{\sec^3\theta\,\tan\theta} = \int \frac{2}{\sec^2\theta}\,d\theta = 2\int \cos^2\theta\,d\theta$

$= 2\int \frac{1+\cos 2\theta}{2}\,d\theta = \theta + \sin\theta\cos\theta + C$

$= \theta + \tan\theta\cos^2\theta + C = \sec^{-1}x + \sqrt{x^2-1}\left(\frac{1}{x^2}\right) + C$

$= \boxed{\sec^{-1}x + \frac{\sqrt{x^2-1}}{x^2} + C}$

(20) $x = 2\sin\theta,\ 0 \le \theta \le \frac{\pi}{6},\ dx = 2\cos\theta\,d\theta$

$(4-x^2)^{3/2} = 8\cos^3\theta$

$\int_0^1 \frac{dx}{(4-x^2)^{3/2}} = \int_0^1 \frac{2\cos\theta\,d\theta}{8\cos^3\theta} = \int_0^1 \frac{1}{4}\sec^2\theta\,d\theta$

$= \frac{1}{4}\tan\theta\Big|_0^{\pi/6} = \frac{1}{4}\left(\frac{\sqrt{3}}{3} - 0\right) = \boxed{\frac{\sqrt{3}}{12}}$

(95) $\int \sqrt{a^2-x^2}\,dx$. Let $x = a\sin\theta \Rightarrow a^2-x^2 = a^2\cos^2\theta$,

$dx = a\cos\theta\, d\theta$.

$$\int |a\cos\theta|\, a\cos\theta\, d\theta = \int a^2\cos^2\theta\, d\theta = a^2\int \frac{1+\cos 2\theta}{2}\, d\theta$$

$$= a^2\left[\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}\right] + C = \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{a}\cdot\frac{\sqrt{a^2-x^2}}{a}\right] + C$$

$$= \boxed{\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2-x^2} + C}$$

(96) $\int \frac{dx}{x^2\sqrt{x^2-a^2}}$ Let $x = a\sec\theta \Rightarrow x^2-a^2 = a^2\tan^2\theta$,

$dx = a\sec\theta\tan\theta\, d\theta$.

$$\int \frac{a\sec\theta\tan\theta\, d\theta}{a^2\sec^2\theta\; a\tan\theta} = \frac{1}{a^2}\int \cos\theta = \frac{1}{a^2}\sin\theta + C$$

$$= \frac{1}{a^2}\tan\theta\cdot\cos\theta + C = \frac{1}{a^2}\frac{\tan\theta}{\sec\theta} + C$$

$$= \frac{1}{a^2}\left[\frac{\sqrt{x^2-a^2}}{a}\Big/(x/a)\right] + C = \boxed{\frac{1}{a^2}\frac{\sqrt{x^2-a^2}}{x} + C}$$

(97) $\int x^n \sin ax\, dx$ Let $u = x^n \Rightarrow du = nx^{n-1}dx$

$dv = \sin ax \Rightarrow v = -\frac{1}{a}\cos ax$

$$\Rightarrow \boxed{\int x^n \sin ax\, dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx}$$

(98) $\int x^n (\ln ax)^m\, dx$ Let $u = (\ln ax)^m \Rightarrow du = \frac{am}{ax}(\ln ax)^{m-1}dx$.

$dv = x^n dx \Rightarrow v = \frac{x^{n+1}}{n+1}$

$$\boxed{\int x^n(\ln ax)^m dx = \frac{(\ln ax)^m x^{n+1}}{n+1} - \frac{m}{(n+1)}\int x^n (\ln ax)^{m-1}\, dx}$$

(99) $\int x^n \sin^{-1}ax\, dx$ $u = \sin^{-1}ax \Rightarrow du = \frac{a}{\sqrt{1-a^2x^2}}dx$

$dv = x^n \Rightarrow v = \frac{x^{n+1}}{n+1}$

$$\boxed{\int x^n \sin^{-1}ax\, dx = \frac{x^{n+1}\sin^{-1}ax}{n+1} - \frac{a}{n+1}\int \frac{x^{n+1}}{\sqrt{1-a^2x^2}}\, dx}$$

(100) $\int x^n \tan^{-1}ax\, dx$ $u = \tan^{-1}ax \Rightarrow du = \frac{a}{1+a^2x^2}dx$

$dv = x^n \Rightarrow v = \frac{x^{n+1}}{n+1}$

$$\boxed{\int x^n \tan^{-1}ax\, dx = \frac{x^{n+1}\tan^{-1}ax}{n+1} - \frac{a}{n+1}\int \frac{x^{n+1}}{1+a^2x^2}\, dx}$$

$\boxed{\S 8.7}$

② $\int_1^3 (2x-1)\,dx$ $\quad (f''=0)$

(I) $T = \frac{b-a}{2(n)}(y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n)$

(a) $T = \frac{2}{8}(1 + 4 + 6 + 8 + 5)$

$= \frac{1}{4}(24) = \boxed{6}$

$|E_T| \leq \frac{M(b-a)^3}{12n^2} = 0.$

(b) $\int_1^3 2x-1\,dx = x^2 - x\Big|_1^3 = 9-3-1+1 = \boxed{6}$

$|E_T| = |6-6| = 0.$

(c) $\frac{|E_T|}{\text{True value}} \cdot 100 = \boxed{0\%}$

(II) $S = \frac{b-a}{3n}(y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{n-2} + 4y_{n-1} + y_n)$

(a) $= \frac{2}{12}(1 + 8 + 6 + 16 + 5)$

$= \frac{1}{6}(36) = \boxed{6}$

$|E_S| \leq \frac{M(b-a)^5}{180n^4} = 0$ $\quad$ b/c $f^{(4)} = 0$

(b) As above, $\int_1^3 2x-1\,dx = 6.$

$|E_S| = 0$

(c) $\frac{|E_S|}{\text{True Value}} \cdot 100 = \boxed{0\%}$

⑧ (I) $\int_2^4 \frac{1}{(s-1)^2}\,ds$

(a) $T = \frac{2}{8}\left(1 + 2\cdot\frac{4}{9} + 2\cdot\frac{1}{4} + 2\cdot\frac{4}{25} + \frac{1}{9}\right) = \frac{1}{4}\left(\frac{141}{50}\right) = \boxed{\frac{141}{200}}$

$|E_T| \leq \frac{M(4-2)^3}{12(16)} = \frac{6(8)}{12(16)} = \boxed{\frac{1}{4}.}$ $\quad f'' = \frac{6}{(s-1)^4}$

(b) $\int_2^4 \frac{1}{(s-1)^2}\,ds = -\frac{1}{(s-1)}\Big|_2^4 = -\frac{1}{3} + 1 = \boxed{\frac{2}{3}}$

$|E_T| = \left|\frac{2}{3} - \frac{141}{200}\right| = \boxed{\frac{23}{600} \approx .0383}$

(c) $\frac{|E_T|}{\text{True Value}} \cdot 100 = \frac{.0383}{2/3} \cdot 100 \approx \boxed{5.75\%}$

$\longrightarrow$

(8) (II) (a) $S = \frac{2}{12}\left(1 + 4\cdot\frac{4}{9} + 2\cdot\frac{1}{4} + 4\cdot\frac{4}{25} + \frac{1}{9}\right) = \frac{1}{6}\left(\frac{1813}{450}\right) = \boxed{\frac{1813}{2700} \approx .6715}$

$|E_S| \le \frac{120\cdot 32}{180\cdot 256} = \boxed{\frac{1}{12}}$ $\qquad f^{(4)} = \frac{120}{(S\cdot 1)^6}$

(b) As above, $\frac{2}{3}$, so

$|E_S| = \left|\frac{1813}{2700} - \frac{2}{3}\right| = \boxed{\frac{13}{2700} \approx .00481}$

(c) $\frac{|E_S|}{\text{True Val}} \cdot 100 \approx \boxed{1\%}$

(16) (a) $M=0$. Then $n=1 \Rightarrow \Delta x = 2 \Rightarrow |E_T| = 0 < 10^{-4}$

(b) $M=0$. Then $n=2$ (n must be even) $\Rightarrow \Delta x = 1 \Rightarrow$
$|E_S| = 0 < 10^{-4}$.

(22) (a) $M=6$. Then $\Delta x = \frac{2}{n} \Rightarrow |E_T| \le \frac{2}{12}\left(\frac{2}{n}\right)^2(6) = \frac{4}{n^2} < 10^{-4}$

$\Rightarrow n^2 > 4(10^4) \Rightarrow n > 200$, so let $\boxed{n=201}$

(b) $M=120$. Then $\Delta x = \frac{2}{n} \Rightarrow |E_S| \le \frac{2}{180}\left(\frac{2}{n}\right)^4(120) = \frac{64}{3n^4} < 10^{-4}$

$\Rightarrow n^4 > \frac{64}{3}(10^4) \Rightarrow n > 21.5$, so let $\boxed{n = 22}$

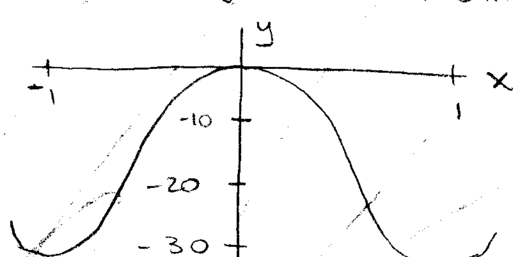
(40) Using a graphing calculator w/n=50: 1.37076

(46) (a) $f'''(x) = -4x^2\cdot 2x\cos(x^2) - 8x\sin(x^2) - 4x\sin(x^2)$

$= -8x^3\cos(x^2) - 12x\sin(x^2)$

$f^{(4)}(x) = (16x^4 - 12)\sin(x^2) - 48x^2\cos(x^2)$

(b)



(c) From the graph, $-30 < f^{(4)}(x) \le 0$ so
$|f^{(4)}(x)| \le 30$ for $-1 \le x \le 1$.

(d) $|E_S| \le \frac{1-(-1)}{180}(\Delta x)^4(30) = \frac{(\Delta x)^4}{3}$

(e) For $0 < \Delta x < 0.4$, $|E_S| \le \frac{(\Delta x)^4}{3} \le \frac{(0.4)^2}{3} \approx .00853 < .01$

(f) $n \ge \frac{1-(-1)}{\Delta x} \ge \frac{2}{0.4} = 5$

(33) (a) $|E_S| \le \frac{b-a}{180}(\Delta x)^4 M$; $n=4 \Rightarrow \Delta x = \frac{\pi}{8}$; $|f^{(4)}| \le 1$

$\Rightarrow M = 1 \Rightarrow |E_S| \le \frac{\pi/2}{180}\left(\frac{\pi}{8}\right)^4(1) \approx \boxed{.00021}$

(b) $\Delta x = \frac{\pi}{8} \Rightarrow \frac{\Delta x}{3} = \frac{\pi}{24}$;

$S = \frac{\pi}{24}(10.47208705) \approx \boxed{1.37079}$

(c) $\frac{.00021}{1.37079} \times 100 \approx \boxed{.015\%}$

(34) (a) $\Delta x = \frac{1}{10}$

$$\operatorname{erf}(1) = \frac{2}{\sqrt{3}}\left(\frac{1}{30}\right)\left(e^0 + 4e^{-1/100} + 2e^{-4/100} + 4e^{-9/100} + 2e^{-16/100} + 4e^{-25/100} + 2e^{-36/100} + 4e^{-49/100} + 2e^{-64/100} + 4e^{-81/100} + e^{-1}\right) \approx \boxed{.843}$$

(b) $|E_S| \le \frac{1}{180}\left(\frac{1}{10}\right)^4(12) \approx \boxed{6.7 \times 10^{-6}}$

(48) $\int_4^\infty \frac{dx}{\sqrt{x}-1}$; $\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}-1}}{\frac{1}{\sqrt{x}}} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}-1} = 1$

and $\int_4^\infty \frac{dx}{\sqrt{x}} = \lim_{b\to\infty} \left[2\sqrt{x}\right]_4^b = \infty$ which diverges,
so $\int_4^\infty \frac{dx}{\sqrt{x}-1}$ diverges also.

(58) $\int_2^\infty \frac{dx}{\ln x}$; $0 < \frac{1}{x} < \frac{1}{\ln x}$ for $x > 2$ and
$\int_2^\infty \frac{1}{x}\,dx$ diverges $\Rightarrow \int_2^\infty \frac{1}{\ln x}\,dx$ diverges.

(60) $\int_{e^e}^\infty \ln(\ln x)\,dx$; Let $x = e^y \Rightarrow y = \ln x$, $dy = \frac{1}{x}\,dx$
$\int_e^\infty (\ln y)e^y\,dy$; $0 < \ln y < (\ln y)e^y$ for $y \geq e$
and $\int_e^\infty \ln y\,dy$ diverges, so
$\int_{e^e}^\infty \ln(\ln x)\,dx$ also diverges.

(65) (a) $\int_1^2 \frac{dx}{x(\ln x)^p}$; $t = \ln x \Rightarrow \int_0^{\ln 2} \frac{dt}{t^p} = \lim_{b\to 0} \frac{1}{-p+1} t^{-p+1} \Big|_b^{\ln 2}$
$= \lim_{b\to 0} \frac{b^{1-p}}{p-1} + \frac{1}{1-p}(\ln 2)^{1-p} \Rightarrow$ converges for $p<1$,
diverges for $p \geq 1$

(b) $\int_2^\infty \frac{dx}{x(\ln x)^p}$; $t = \ln x \Rightarrow \int_{\ln 2}^\infty \frac{dt}{t^p}$
Converges for $p>1$, diverges for $p \leq 1$.

(66) $\int_0^\infty \frac{2x\,dx}{x^2+1} = \lim_{b\to\infty} \left[\ln(x^2+1)\right]_0^b = \lim_{b\to\infty} \left[\ln(b^2+1) - 0\right] = \infty$
$\Rightarrow$ the integral diverges.
But $\lim_{b\to\infty} \int_{-b}^b \frac{2x\,dx}{x^2+1} = \lim_{b\to\infty} \left[\ln(x^2+1)\right]_{-b}^b$
$= \ln(b^2+1) - \ln(b^2+1) = 0$.

(86) $\int_{-\infty}^\infty \frac{dx}{(x+1)^2} = \int_{-\infty}^{-2} \frac{dx}{(x+1)^2} + \int_{-2}^{-1} \frac{dx}{(x+1)^2} + \int_{-1}^{2} \frac{dx}{(x+1)^2} + \int_2^\infty \frac{dx}{(x+1)^2}$.
$\lim_{b\to -1} \int_{-2}^b \frac{dx}{(x+1)^2} = -\lim_{b\to -1} \left[\frac{1}{x+1}\right]_{-2}^b = \infty$ diverges

HW 10

§8.5

(2) $\int \frac{3\,dy}{\sqrt{1+9y^2}}$ Let $x=3y \Rightarrow dx = 3\,dy$

$\int \frac{dx}{\sqrt{1+x^2}}$ $x = \tan t \Rightarrow dx = \sec^2 t = \frac{1}{\cos^2 t}\,dt$

$\int \frac{1}{\cos^2 t\sqrt{1+\tan^2 t}}\,dt = \int \frac{1}{\cos^2 t \cdot \sec t} = \int \frac{1}{\cos t}\,dt$

$= \int \sec t\,dt = \ln|\sec t + \tan t| + C$

$= \ln\left|\sqrt{1+x^2} + x\right| + C = \boxed{\ln\left|\sqrt{1+9y^2} + 3y\right| + C}$

(8) $t = \frac{1}{3}\sin\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$; $dt = \frac{1}{3}\cos\theta\,d\theta$,

$\sqrt{1-9t^2} = \cos\theta$

$\int\sqrt{1-9t^2}\,dt = \frac{1}{3}\int\cos^2\theta\,d\theta = \frac{1}{6}(\theta + \sin\theta\cos\theta) + C$

$= \boxed{\frac{1}{6}\left[\sin^{-1}(3t) + 3t\sqrt{1-9t^2}\right] + C}$

(14) $x = \sec\theta$, $0 < x < \pi/2$, $dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$\int \frac{2\,dx}{x^3\sqrt{x^2-1}} = \int \frac{2\sec\theta\tan\theta\,d\theta}{\sec^3\theta\,\tan\theta} = \int \frac{2}{\sec^2\theta}\,d\theta = 2\int\cos^2\theta\,d\theta$

$= 2\int \frac{1+\cos 2\theta}{2}\,d\theta = \theta + \sin\theta\cos\theta + C$

$= \theta + \tan\theta\cos^2\theta + C = \sec^{-1}x + \sqrt{x^2-1}\left(\frac{1}{x^2}\right) + C$

$= \boxed{\sec^{-1}x + \frac{\sqrt{x^2-1}}{x^2} + C}$

(20) $x = 2\sin\theta$, $0 \le \theta \le \frac{\pi}{6}$, $dx = 2\cos\theta\,d\theta$

$(4-x^2)^{3/2} = 8\cos^3\theta$

$\int_0^1 \frac{dx}{(4-x^2)^{3/2}} = \int_0^1 \frac{2\cos\theta\,d\theta}{8\cos^3\theta} = \int_0^1 \frac{1}{4}\sec^2\theta\,d\theta$

$= \frac{1}{4}\tan\theta\Big|_0^{\pi/6} = \frac{1}{4}\left(\frac{\sqrt{3}}{3} - 0\right) = \boxed{\frac{\sqrt{3}}{12}}$

(30) Let $e^t = \tan\theta$; $t = \ln(\tan\theta)$; $\tan^{-1}(\frac{3}{4}) \le \theta \le \tan^{-1}(\frac{4}{3})$.

$dt = \cot\theta \sec^2\theta \, d\theta$; $1+e^{2t} = 1+\tan^2\theta = \sec^2\theta$.

$$\int_{\ln(3/4)}^{\ln(4/3)} \frac{e^t\,dt}{(1+e^{2t})^{3/2}} = \int_{\tan^{-1}(3/4)}^{\tan^{-1}(4/3)} \frac{\tan\theta\cot\theta\sec^2\theta}{\sec^3\theta} = \int_{\tan^{-1}(3/4)}^{\tan^{-1}(4/3)} \cos\theta \, d\theta$$

$$= \sin\theta \Big|_{\tan^{-1}(3/4)}^{\tan^{-1}(4/3)} = \frac{4}{5} - \frac{3}{5} = \boxed{\frac{1}{5}}$$

(32) $y = e^{\tan\theta}$, $0 \le \theta \le \frac{\pi}{4}$, $dy = \sec^2\theta \, e^{\tan\theta} \, d\theta$

$\sqrt{1+(\ln y)^2} = \sec\theta$

$$\int_1^e \frac{dy}{y\sqrt{1+(\ln y)^2}} = \int_0^{\pi/4} \frac{\sec^2\theta \, e^{\tan\theta}}{e^{\tan\theta}\sec\theta} d\theta = \int_0^{\pi/4} \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| \Big|_0^{\pi/4} = \ln|\sqrt{2}+1| - \ln|1+0|$$

$$= \boxed{\ln(1+\sqrt{2})}$$

(38) $\sqrt{x^2-9}\,\frac{dy}{dx} = 1$, $dy = \frac{dx}{\sqrt{x^2-9}}$; $y = \int \frac{dx}{\sqrt{x^2-9}}$.

Let $x = 3\sec\theta$, $0 < \theta < \pi/2$, $dx = 3\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-9} = 3\tan\theta$

$$y = \int \frac{3\sec\theta\tan\theta\,d\theta}{3\tan\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$$

$x = 5$ and $y = \ln 3 \Rightarrow \ln 3 = \ln 3 + C \Rightarrow C = 0$

$$\Rightarrow \boxed{y = \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right|}$$

(40) $(x^2+1)^2 \frac{dy}{dx} = \sqrt{x^2+1}$, $dy = \frac{dx}{(x^2+1)^{3/2}}$; $x = \tan\theta$, $dx = \sec^2\theta\,d\theta$.

$(x^2+1)^{3/2} = \sec^3\theta$.

$$y = \int \frac{\sec^2\theta\,d\theta}{\sec^3\theta} = \int \cos\theta\,d\theta = \sin\theta + C$$

$$= \tan\theta\cos\theta + C = \frac{\tan\theta}{\sec\theta} + C = \frac{x}{\sqrt{x^2+1}} + C$$

$x = 0$ and $y = 1 \Rightarrow 1 = 0 + C \Rightarrow C = 1$

So $\boxed{y = \frac{x}{\sqrt{x^2+1}} + 1}$

(50) $\int \frac{\cos t \, dt}{1-\cos t} = \int \frac{\frac{1-z^2}{1+z^2}\left(\frac{2\,dz}{1+z^2}\right)}{1-\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{2(1-z^2)\,dz}{(1+z^2)^2-(1-z^2)(1+z^2)}$

$= \int \frac{2(1-z^2)\,dz}{(1+z^2)(1+z^2-1+z^2)} = \int \frac{2(1-z^2)\,dz}{(1+z^2)(2z^2)} = \int \frac{1-z^2}{z^2(1+z^2)}\,dz$

$= \int \frac{dz}{z^2(1+z^2)} - \int \frac{dz}{1+z^2} = \int \frac{dz}{z^2} - \int \frac{dz}{(1+z^2)} - \int \frac{dz}{(1+z^2)} \rightarrow$ Partial fractions

$= \int \frac{dz}{z^2} - 2\int \frac{dz}{(1+z^2)}$

$= -\frac{1}{z} - 2\tan^{-1}(z) + C = \boxed{-\cot\left(\frac{t}{2}\right) - t + C}$

(52) $\int \csc\theta \, d\theta = \int \frac{1}{\sin\theta}\,d\theta = \int \left(\frac{2dz}{1+z^2}\right)\left(\frac{1+z^2}{2z}\right)$

$= \int \frac{dz}{z} = \ln|z| + C = \boxed{\ln\left|\tan\frac{\theta}{2}\right| + C}$

§8.6

(93) Let $u = ax+b \Rightarrow du = a\,dx$ and $x = \frac{u-b}{a}$.

$\int \frac{u-b}{a} \cdot \frac{1}{u^2} \cdot \frac{1}{a}\,du = \int \frac{u-b}{a^2} \cdot \frac{1}{u^2}\,du$

$= \frac{1}{a^2}\int \frac{1}{u} - \frac{b}{u^2}\,du = \frac{1}{a^2}\left(\ln|u| + \frac{b}{u}\right) + C$

$= \boxed{\frac{1}{a^2}\left[\ln|ax+b| + \frac{b}{ax+b}\right] + C}$

(94) $\int \frac{dx}{(a^2+x^2)^2}$. Let $x = a\tan\theta \Rightarrow a^2+x^2 = a^2\sec^2\theta$,

$dx = a\sec^2\theta\,d\theta$.

$\int \frac{a\sec^2\theta\,d\theta}{a^4\sec^4\theta} = \int \frac{1}{a^3} \cdot \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{a^3}\int \cos^2\theta\,d\theta$

$= \frac{1}{a^3}\int \frac{1+\cos 2\theta}{2}\,d\theta = \frac{1}{a^3}\left(\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}\right) + C$

$= \frac{1}{a^3}\left(\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2} + \frac{\tan\theta}{2\sec^2\theta}\right) = \frac{1}{2a^3}\left(\tan^{-1}\left(\frac{x}{a}\right) + \left(\frac{x}{a}\right)\left(\frac{a^2}{a^2+x^2}\right)\right) + C$

$= \frac{1}{2a^3}\left(\tan^{-1}\left(\frac{x}{a}\right) + \frac{ax}{a^2+x^2}\right) + C$

$= \boxed{\frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) + \frac{x}{2a^2(a^2+x^2)} + C}$

(95) $\int \sqrt{a^2-x^2}\ dx$. Let $x = a\sin\theta \Rightarrow a^2-x^2 = a^2\cos^2\theta$,

$dx = a\cos\theta\, d\theta$.

$\int |a\cos\theta|\, a\cos\theta\, d\theta = \int a^2\cos^2\theta\, d\theta = a^2\int \frac{1+\cos 2\theta}{2}\, d\theta$

$= a^2\left[\frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2}\right] + C = \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{a}\cdot\frac{\sqrt{a^2-x^2}}{a}\right] + C$

$= \boxed{\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2-x^2} + C}$

(96) $\int \frac{dx}{x^2\sqrt{x^2-a^2}}$ Let $x = a\sec\theta \Rightarrow x^2-a^2 = a^2\tan^2\theta$,

$dx = a\sec\theta\tan\theta\, d\theta$.

$\int \frac{a\sec\theta\tan\theta\, d\theta}{a^2\sec^2\theta\ a\tan\theta} = \frac{1}{a^2}\int \cos\theta = \frac{1}{a^2}\sin\theta + C$

$= \frac{1}{a^2}\tan\theta\cdot\cos\theta + C = \frac{1}{a^2}\frac{\tan\theta}{\sec\theta} + C$

$= \frac{1}{a^2}\left[\frac{\sqrt{x^2-a^2}}{a}\Big/(x/a)\right] + C = \boxed{\frac{1}{a^2}\frac{\sqrt{x^2-a^2}}{x} + C}$

(97) $\int x^n \sin ax\ dx$ Let $u = x^n \Rightarrow du = nx^{n-1}dx$

$dv = \sin ax \Rightarrow v = -\frac{1}{a}\cos ax$

$\Rightarrow \boxed{\int x^n \sin ax\ dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\ dx}$

(98) $\int x^n (\ln ax)^m\, dx$ Let $u = (\ln ax)^m \Rightarrow du = \frac{am}{ax}(\ln ax)^{m-1}dx$.

$dv = x^n dx \Rightarrow v = \frac{x^{n+1}}{n+1}$

$\boxed{\int x^n(\ln ax)^m dx = \frac{(\ln ax)^m x^{n+1}}{n+1} - \frac{m}{(n+1)}\int x^n(\ln ax)^{m-1}\, dx}$

(99) $\int x^n \sin^{-1} ax\ dx$ $u = \sin^{-1}ax \Rightarrow du = \frac{a}{\sqrt{1-a^2x^2}}dx$

$dv = x^n \Rightarrow v = \frac{x^{n+1}}{n+1}$

$\boxed{\int x^n \sin^{-1}ax\ dx = \frac{x^{n+1}\sin^{-1}ax}{n+1} - \frac{a}{n+1}\int\frac{x^{n+1}}{\sqrt{1-a^2x^2}}\, dx}$

(100) $\int x^n \tan^{-1} ax\ dx$ $u = \tan^{-1}ax \Rightarrow du = \frac{a}{1+a^2x^2}dx$.

$dv = x^n \Rightarrow v = \frac{x^{n+1}}{n+1}$

$\boxed{\int x^n \tan^{-1}ax\ dx = \frac{x^{n+1}\tan^{-1}ax}{n+1} - \frac{a}{n+1}\int\frac{x^{n+1}}{1+a^2x^2}\, dx}$