21 B - HW 1 Solutions

$\boxed{\S 4.8}$

4c) $F(x) = \frac{1}{2}x^{-2} + \frac{1}{2}x^{2} - x + C$

Check: $F'(x) = -x^{-3} + x - 1$ ✓

7c) $F(x) = \frac{2}{3}x^{3/2} + 2x^{1/2} + C$

Check: $F'(x) = x^{1/2} + x^{-1/2}$ ✓

53) $F(x) = -e^{-x} + \frac{4^x}{\ln 4} + C$

Check: $F'(x) = e^{-x} + 4^x$ ✓

87) $\frac{dy}{dx} = 2x \Rightarrow y = x^2 + C$

$y(1) = 1^2 + C = 4 \Rightarrow C = 3$

$y = x^2 + 3$

(b) is the correct graph.

88) $\frac{dy}{dx} = -x \Rightarrow y = -\frac{1}{2}x^2 + C$

$y(-1) = -\frac{1}{2}(-1)^2 + C = \underline{1}$

$\Rightarrow -\frac{1}{2} + C = \underline{1}$

$\Rightarrow C = 3/2$

So $y = -\frac{1}{2}x^2 + \frac{3}{2}$

(b) is the correct graph.

111) $\frac{dy}{dx} = 3\sqrt{x} = 3x^{1/2}$

$\Rightarrow \quad y = \frac{3}{(3/2)} x^{3/2} + C = 2x^{3/2} + C$

$4 = 2(9)^{3/2} + C = 2(27) + C = 54 + C$

$\Rightarrow \quad C = -50$

So $\quad y = 2x^{3/2} - 50.$

123) $\frac{d^2s}{dt^2} = a \quad \Longrightarrow \quad \frac{ds}{dt} = at + C$

First initial condition: $\frac{ds}{dt} = v_0$ for $t = 0$,

So $\quad \frac{ds}{dt} = at + C$ and $\frac{ds}{dt} = v_0$ at $t = 0$

$\Rightarrow \quad a(0) + C = v_0 \quad \Rightarrow \quad C = v_0.$

So $\quad \frac{ds}{dt} = at + v_0.$

Then $\quad s(t) = \frac{1}{2}at^2 + v_0 t + C.$

Second initial condition: $s(t) = s_0$ for $t = 0$,

So $\quad s(0) = \frac{1}{2}a(0)^2 + v_0(0) + C = s_0$

$\Rightarrow \quad C = s_0.$

$\therefore \quad s(t) = \frac{1}{2}at^2 + v_0 t + s_0.$

$\boxed{\S 5.1}$

1) (a) $\text{Area} = f(0)\cdot\frac{1}{2} + f(\frac{1}{2})\cdot(\frac{1}{2})$

$= 0\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{2} = 0 + \frac{1}{8}$

$= \frac{1}{8}$ units^2

(b) $\text{Area} = f(0)\cdot\frac{1}{4} + f(\frac{1}{4})\cdot\frac{1}{4} + f(\frac{1}{2})\cdot\frac{1}{4} + f(\frac{3}{4})\cdot\frac{1}{4}$

$= 0 + \frac{1}{64} + \frac{1}{16} + \frac{9}{64}$

$= \frac{7}{32}$ units^2

(c) $\text{Area} = f(\frac{1}{2})\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2}$

$= \frac{1}{8} + \frac{1}{2}$

$= \frac{5}{8}$ units^2

(d) $\text{Area} = f(\frac{1}{4})\cdot\frac{1}{4} + f(\frac{1}{2})\cdot\frac{1}{4} + f(\frac{3}{4})\cdot\frac{1}{4} + f(1)\cdot\frac{1}{4}$

$= \frac{1}{64} + \frac{1}{16} + \frac{9}{64} + \frac{1}{4}$

$= \frac{15}{32}$ units^2

5) Area (2 rect.) $= f(\frac{1}{4})\cdot\frac{1}{2} + f(\frac{3}{4})\cdot\frac{1}{2} = \frac{1}{32} + \frac{9}{32}$

$= \frac{10}{32} = \frac{5}{16}$ units^2

Area (4 rect.) $= f(\frac{1}{8})\cdot\frac{1}{4} + f(\frac{3}{8})\cdot\frac{1}{4} + f(\frac{5}{8})\cdot\frac{1}{4} + f(\frac{7}{8})\cdot\frac{1}{4}$

$= \frac{1}{256} + \frac{9}{256} + \frac{25}{256} + \frac{49}{256} = \frac{84}{256}$

$= \frac{21}{64}$ units^2

12) Area $\approx 20(.001) + 50(.001) + 72(.001) + 90(.001) + 102(.001)$

$+ 112(.001) + 120(.001) + 128(.001) + 134(.001)$

$+ 139(.001)$

$= 0.967$ miles

(b) Halfway point is .4835 miles

$20(.001) + 50(.001) + 72(.001) + 90(.001) + 102(.001) + 112(.001)$

$= .446,$

So halfway point is just over .006 hrs

$\Rightarrow$ car was going just over 116 mph ($\approx$ 120 mph).

14) $V_0 = 400$ ft/sec

$g = 32$ ft/sec^2

So we have

t	0	1	2	3	4	5
$v(t)$	400	368	336	304	272	240

Then $v(5) \approx 240$ ft/sec

(b) Lower est $\Rightarrow$ right endpts since $v(t)$ is decreasing.

So height $\approx 368(1) + 336(1) + 304(1) + 272(1) + 240(1)$

$= 1520$ ft.

16) Area $= f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2$

$= 1 + 1/2 + 1/3 + 1/4 = \frac{25}{12} = 2\frac{1}{12}$ units^2

Avg. Area $= \frac{(25/12)}{8} = \frac{25}{12}\cdot\frac{1}{8} = \frac{25}{96}$ units^2.

§ 5.2

8) (a) $\sum_{k=1}^{6} (-2)^{k-1} = 1 - 2 + 4 - 8 + 16 - 32$

(b) $\sum_{k=0}^{5} (-1)^k (2)^k = 1 - 2 + 4 - 8 + 16 - 32$

(c) $\sum_{k=-2}^{3} (-1)^{k+1} 2^{k+2} = -1 + 2 - 4 + 8 - 16 + 32$

$\rightarrow$ So both (a) and (b)

9) Both (a) and (c) are equivalent;

(b) is not.

10) Both (a) and (c) are equivalent;

(b) is not.

18) (a) $8 \cdot 0 = 0$

(b) $250 \cdot 1 = 250$

(c) $0 + n \cdot 1 = n$

(d) $1 - n \cdot 1 = 1 - n$

25) $\sum_{k=1}^{5} k(3k+5) = \sum_{k=1}^{5} 3k^2 + 5k = \sum_{k=1}^{5} 3k^2 + \sum_{k=1}^{5} 5k$

$= 3 \sum_{k=1}^{5} k^2 + 5 \sum_{k=1}^{5} k$

$= 3 \left[\frac{5(5+1)(2 \cdot 5+1)}{6} \right] + 5 \left[\frac{5(6)}{2} \right]$

$= 3 \left[\frac{5 \cdot 6 \cdot 11}{6} \right] + 5 \left[\frac{30}{2} \right] = 3(55) + 5(15)$

$= 240.$