# 21 B - HW 2 Solutions

## § 5.3

(5) $\int_2^3 \frac{1}{1-x}\,dx$

(8) $\int_0^{\pi/4} \tan x \, dx$

(9) a) $\int_2^2 g(x)\,dx = 0$

b) $\int_5^1 g(x)\,dx = -8$

c) $\int_1^2 3f(x)\,dx = -12$

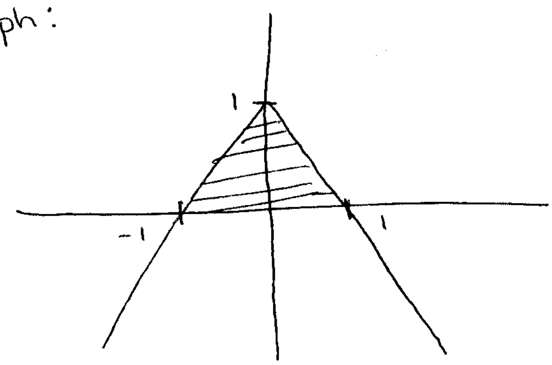
d) $\int_2^5 f(x)\,dx = 6-(-4) = 10$

e) $\int_1^5 [f(x)-g(x)]\,dx = 6-8 = -2$

f) $\int_1^5 [4f(x)-g(x)]\,dx = 24-8 = 16$

(20) $\int_{-1}^1 (1-|x|)\,dx$

Graph:



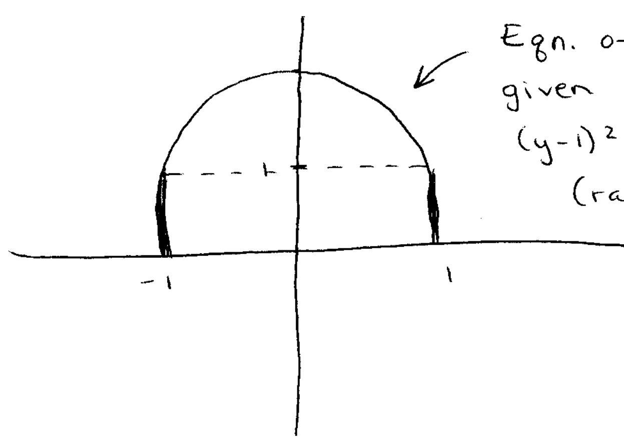

Area of triangle =

$\frac{1}{2}$(base)(height)

$= \frac{1}{2}(2)(1)$

$= 1.$

(22) $\int_{-1}^1 (1+\sqrt{1-x^2})\,dx$




Eqn. of semicircle given by $(y-1)^2+x^2=1$ (radius = 1)

Semicircle Area $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(1)^2$

$= \frac{\pi}{2}$

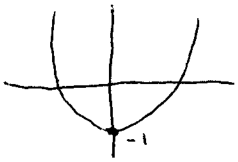
Rectangle Area = (length)(width)

$= (2)(1)$

$= 2$

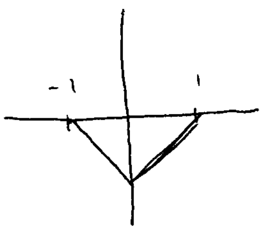
Total Area $= 2 + \frac{\pi}{2}.$

(55) Avg. Value $= \dfrac{1}{\sqrt{3}} \displaystyle\int_0^{\sqrt{3}} x^2-1 \, dx$




$$= \frac{1}{\sqrt{3}} \left( \frac{1}{3}x^3 - x \Big|_0^{\sqrt{3}} \right) = \frac{1}{\sqrt{3}} \left( \sqrt{3} - \sqrt{3} \right) = \boxed{0}$$

(61) (a) Avg. Value $= \dfrac{1}{2} \displaystyle\int_{-1}^{1} |x|-1 \, dx$

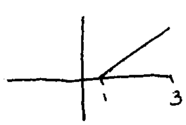



$$= \frac{1}{2}\left[ \int_{-1}^{0} -x-1 \, dx + \int_0^1 x-1 \, dx \right]$$

$$= \frac{1}{2}\left( -\frac{x^2}{2} - x \Big|_{-1}^{0} + \frac{x^2}{2} - x \Big|_0^1 \right)$$

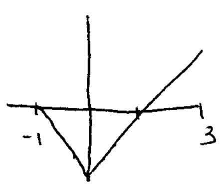
$$= \frac{1}{2}\left[ \left(0 - \left(-\tfrac{1}{2}+1\right)\right) + \tfrac{1}{2} - 1 - 0 \right] = \frac{1}{2}(-1) = \boxed{\frac{-1}{2}}$$

(b) Avg. Value $= \dfrac{1}{2} \displaystyle\int_1^3 x-1 \, dx$




$$= \frac{1}{2}\left( \frac{x^2}{2} - x \Big|_1^3 \right) = \frac{1}{2}\left[ \frac{9}{2} - 3 - \frac{1}{2} + 1 \right] = \boxed{1}$$

(c) Avg. Value $= \dfrac{1}{4} \displaystyle\int_{-1}^{3} |x|-1 \, dx = \frac{1}{4}\left[ \int_{-1}^{1} |x|-1 \, dx + \int_1^3 |x|-1 \, dx \right]$




$$= \frac{1}{4}(-1+2) = \boxed{\frac{1}{4}}$$

(68) $\displaystyle\int_0^1 \sqrt{x+8} \, dx$

$$f_{min}(b-a) \leq \int_a^b f(x)\,dx \leq f_{max}(b-a)$$

$$\Rightarrow \sqrt{8}\,(1-0) \leq \int_0^1 \sqrt{x+8}\,dx \leq \sqrt{9}\,(b-0)$$

$$\Rightarrow \boxed{2\sqrt{2} \leq \int_0^1 \sqrt{x+8}\,dx \leq 3}$$

(71) $\displaystyle\int_0^1 \sin x \, dx \leq \int_0^1 x \, dx = \frac{1}{2}x^2 \Big|_0^1 = \boxed{\frac{1}{2}}$

(72) $\displaystyle\int_0^1 \sec x \, dx \geq \int_0^1 1 + \frac{x^2}{2} \, dx = x + \frac{x^3}{6} \Big|_0^1 = \boxed{\frac{7}{6}}$

§ 5.4

(6) $\int_0^5 x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^5 = \frac{2}{5}(5)^{5/2} - 0 = \frac{2}{5}(25)\sqrt{5} = \boxed{10\sqrt{5}}$

(10) $\int_0^\pi (1+\cos x)\,dx = x + \sin x\Big|_0^\pi = \pi - 1 - 0 = \boxed{\pi - 1}$

(16) $\int_{-\pi/3}^{\pi/3} \frac{1}{2} - \frac{1}{2}\cos 2t\,dt = \frac{1}{2}t - \frac{1}{4}\sin 2t\Big|_{-\pi/3}^{\pi/3} = \boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{4}}$

(28) $\int_1^2 \left(\frac{1}{x} - e^{-x}\right)dx = \ln x + e^{-x}\Big|_1^2 = \ln 2 + \frac{1}{e^2} - \frac{1}{e}.$

(32) $\int_{-1}^0 \pi^{x-1}\,dx = \frac{\pi^{x-1}}{\ln\pi}\Big|_{-1}^0 = \boxed{\frac{1}{\pi\ln\pi}\left(1 - \frac{1}{\pi}\right)}$

(58) $\left[\int_{\pi/6}^{5\pi/6} \sin x\,dx\right] - \left[\left(\frac{4\pi}{6}\cdot\sin\frac{\pi}{6}\right)\right]$

$= -\cos x\Big|_{\pi/6}^{5\pi/6} - \frac{4\pi}{12}$

$= \boxed{\sqrt{3} - \frac{\pi}{3}}$

(60) Area $= 2\left(1+\frac{\pi}{4}\right) - \left[\int_{-\pi/4}^0 \sec^2 t\,dt + \int_0^1 1-t^2\,dt\right]$

$= 2 + \frac{\pi}{2} - \left[\tan t\Big|_{-\pi/4}^0 + \left(t - \frac{1}{3}t^3\Big|_0^1\right)\right]$

$= 2 + \frac{\pi}{2} - \left(1 + 2/3\right)$

$= 2 + \frac{\pi}{2} - \frac{5}{3} = \boxed{\frac{\pi}{2} + \frac{1}{3}}$