

8.1 - 2, 4, 8, 12, 30, 38, 40, 44, 50, 54, 58, 64

$$\textcircled{2} \int \frac{3 \cos x \, dx}{\sqrt{1+3 \sin x}}$$

$$\text{Let } u = 1 + 3 \sin x \Rightarrow du = 3 \cos x \, dx.$$

$$\int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = 2u^{1/2} + C$$
$$= \boxed{2\sqrt{1+3 \sin x} + C}$$

$$\textcircled{4} \int \cot^3 y \csc^2 y \, dy$$

$$\text{Let } u = \cot y \Rightarrow du = -\csc^2 y \, dy.$$

$$\int -u^3 du = -\frac{1}{4}u^4 + C$$
$$= \boxed{-\frac{1}{4} \cot^4 y + C}$$

$$\textcircled{8} \int \frac{dx}{x-\sqrt{x}} = \int \frac{dx}{\sqrt{x}(\sqrt{x}-1)}$$

$$\text{Let } u = \sqrt{x}-1 \Rightarrow du = \frac{1}{2\sqrt{x}} dx$$

$$\int 2 \cdot \frac{1}{u} du = 2 \ln|u| + C$$
$$= \boxed{2 \ln|\sqrt{x}-1| + C}$$

$$\textcircled{12} \int \frac{\cot(3+\ln x)}{x} dx$$

$$\text{Let } u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\int \cot(3+u) du = \ln|\sin(3+u)| + C$$
$$= \boxed{\ln|\sin(3+\ln x)| + C}$$

$$\textcircled{30} \int \frac{2 dx}{x\sqrt{1-4 \ln^2 x}}$$

$$\text{Let } u = 2 \ln x \Rightarrow du = \frac{2}{x} dx$$

$$\int \frac{1}{\sqrt{1-u^2}} du = \sin^{-1}\left(\frac{u}{1}\right) + C$$
$$= \boxed{\sin^{-1}(2 \ln x) + C}$$

$$(38) \int_2^4 \frac{2 dx}{x^2 - 6x + 10}$$

CTS: $x^2 - 6x + 10 = (x^2 - 6x + 9) + 1 = (x-3)^2 + 1$

$$\int_2^4 \frac{2 dx}{(x-3)^2 + 1}$$

Let $u = x-3 \Rightarrow du = dx$

$$\int_2^4 \frac{2}{u^2 + 1} du = 2 \cdot \frac{1}{1} \tan^{-1}\left(\frac{u}{1}\right) \Big|_2^4$$

$$= 2 \tan^{-1}(x-3) \Big|_2^4 = 2 [\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= 2 \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) \right) = 2 \left(\frac{\pi}{2} \right) = \boxed{\pi}$$

$$(40) \int \frac{d\theta}{\sqrt{2\theta - \theta^2}}$$

CTS: $2\theta - \theta^2 = -(\theta^2 - 2\theta + 1) + 1 = 1 - (\theta-1)^2$

$$\int \frac{d\theta}{\sqrt{1 - (\theta-1)^2}}$$

Let $u = \theta-1 \Rightarrow du = d\theta$

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}\left(\frac{u}{1}\right) + C = \boxed{\sin^{-1}(\theta-1) + C}$$

$$(44) \int (\csc x - \tan x)^2 dx$$

$$(\csc x - \tan x)^2 = \csc^2 x - 2 \csc x \tan x + \tan^2 x$$

$$= \csc^2 x - 2 \left(\frac{1}{\sin x} \right) \left(\frac{\sin x}{\cos x} \right) + (\sec^2 x - 1)$$

$$= \csc^2 x - 2 \sec x + (\sec^2 x - 1)$$

$$\int \csc^2 x dx - 2 \int \sec x + \int (\sec^2 x - 1) dx$$

$$= \boxed{-\cot x - 2 \ln |\sec x + \tan x| + \tan x - x + C}$$

$$(50) \int_{-1}^3 \frac{4x^2 - 7}{2x+3} dx$$

$$\int_{-1}^3 (2x-3) dx + \int_{-1}^3 \frac{2}{2x+3} dx$$

$$x^2 - 3x \Big|_{-1}^3 + \int_{-1}^3 \frac{1}{u} du \quad (u=2x+3, du=2 dx)$$

$$(9-9) - (1+3) + \ln |2x+3| \Big|_{-1}^3$$

$$-4 + \ln(9) - \ln(1)$$

$$-4 + \ln(9) - 0 = -4 + \ln 3^2 = \boxed{-4 + 2 \ln 3}$$

$$\text{Division: } \begin{array}{r} 2x-3 \\ 2x+3 \overline{) 4x^2-7} \\ \underline{-4x^2+6x} \\ -6x-7 \\ \underline{+6x+9} \\ 2 \end{array}$$

$$\begin{aligned}
 (54) \quad \int \frac{x+2\sqrt{x-1}}{2x\sqrt{x-1}} dx &= \int \frac{x}{2x\sqrt{x-1}} dx + \int \frac{2\sqrt{x-1}}{2x\sqrt{x-1}} \\
 &= \int \frac{1}{2\sqrt{x-1}} dx + \int \frac{1}{x} dx \\
 (u=x-1, du=dx) &= \int \frac{1}{2\sqrt{u}} du + \ln|x| + C \\
 &= \sqrt{u} + \ln|x| + C \\
 &= \boxed{\sqrt{x-1} + \ln|x| + C}
 \end{aligned}$$

$$\begin{aligned}
 (58) \quad \int \frac{1}{1+\cos x} dx \\
 \text{Note: } 1+\cos x &= 1+\cos\left(2 \cdot \frac{x}{2}\right) = 2\cos^2\left(\frac{x}{2}\right), \text{ so} \\
 \int \frac{1}{1+\cos x} dx &= \int \frac{1}{2\cos^2\left(\frac{x}{2}\right)} dx \\
 &= \frac{1}{2} \int \sec^2\left(\frac{x}{2}\right) dx \\
 &= \boxed{\frac{1}{2} \tan\left(\frac{x}{2}\right) + C}
 \end{aligned}$$

$$\begin{aligned}
 (64) \quad \int_0^\pi \sqrt{1-\cos 2x} dx \\
 \text{Recall: } 1+\cos 2x &= 2\cos^2 x \\
 \Rightarrow -\cos 2x &= 1-2\cos^2 x \\
 \int_0^\pi \sqrt{1+1-2\cos^2 x} dx &= \int_0^\pi \sqrt{2-2\cos^2 x} dx \\
 &= \int_0^\pi \sqrt{2} \cdot \sqrt{1-\cos^2 x} dx = \int_0^\pi \sqrt{2} \sqrt{\sin^2 x} dx \\
 &= \sqrt{2} \int_0^\pi |\sin x| dx = \sqrt{2} \int_0^\pi \sin x dx \quad \left(\begin{array}{l} \text{b/c } \sin x \geq 0 \\ \text{for } 0 \leq x \leq \pi \end{array}\right) \\
 &= \sqrt{2} (-\cos x \Big|_0^\pi) = \sqrt{2} [1 - (-1)] = \boxed{2\sqrt{2}}
 \end{aligned}$$

