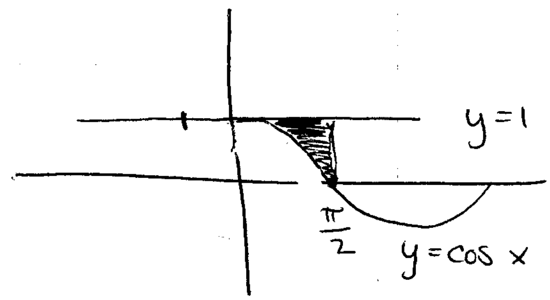
21 B Midterm 1 Solutions

① $y=1$, $x=\pi/2$
$y=\cos x$



$$\int_0^{\pi/2} 1-\cos x \, dx$$
$$= x-\sin x \Big|_0^{\pi/2}$$
$$= \frac{\pi}{2}-1-0+0 = \frac{\pi}{2}-1.$$

So Area = $\boxed{\frac{\pi}{2}-1}$

② (i) $\int_1^2 \frac{1}{x}+e^{-x} \, dx = \ln x - e^{-x} \Big|_1^2 = \ln 2 - e^{-2} - [0-e^{-1}]$
$= \boxed{\ln 2 - e^{-2}+e^{-1}}$

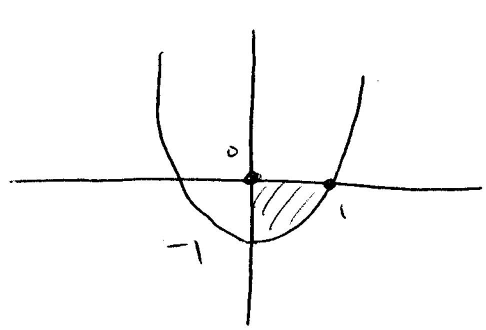
(ii) $\int_{-\pi/2}^{\pi/2} (1-\cos 2t)\, dt = t - \frac{1}{2}\sin 2t \Big|_{-\pi/2}^{\pi/2}$
$= \left(\frac{\pi}{2}-\frac{1}{2}\sin\pi\right) - \left(-\frac{\pi}{2}-\frac{1}{2}\sin(-\pi)\right)$
$= \frac{\pi}{2} - \frac{1}{2}(0) + \frac{\pi}{2} + \frac{1}{2}(0)$
$= \boxed{\pi}$

③ (i) $\int_0^x t^{3/2}\, dt = \frac{2}{5}t^{5/2}\Big|_0^x = \frac{2}{5}x^{5/2} - 0 = \boxed{\frac{2}{5}x^{5/2}}$

(ii) $\int_{-1}^x \pi^{t-1}\, dt = \frac{1}{\ln\pi}\pi^{t-1}\Big|_{-1}^x = \boxed{\frac{\pi^{x-1}}{\ln\pi} - \frac{1}{\pi^2\ln\pi}}$

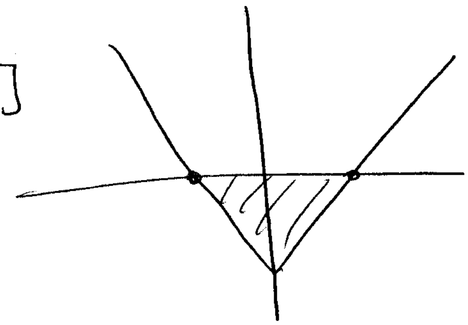
④ $\int_0^5 \sqrt{x+4}\, dx$
$f_{min}(5-0) \le \int \le f_{max}(5-0)$
$\Rightarrow 2(5) \le \int \le 3(5)$
$10 \le \int_0^5 \sqrt{x+4}\, dx \le 15$

⑤ (i) $f(x) = x^2-1$
$\frac{1}{1-0}\int_0^1 x^2-1\, dx$
$= \int_0^1 x^2-1\, dx = \frac{1}{3}x^3 - x\Big|_0^1$
$= \frac{1}{3}-1$
$= \boxed{-2/3}$



5 ii) $f(x) = |x| - 1$ on $[-1,1]$

$$\frac{1}{2}\left[\int_{-1}^{0} -x-1\,dx + \int_{0}^{1} x-1\,dx\right]$$



$$= \frac{1}{2}\left[-\frac{1}{2}x^2 - x\Big|_{-1}^{0} + \frac{1}{2}x^2 - x\Big|_{0}^{1}\right]$$

$$= \frac{1}{2}\left[\left(0 - \left(-\frac{1}{2}+1\right)\right) + \frac{1}{2} - 1\right]$$

$$= \frac{1}{2}\left[-\frac{1}{2} - \frac{1}{2}\right] = \frac{1}{2}(-1) = \boxed{-\frac{1}{2}}$$

6 $\frac{d^2s}{dt^2} = a$ $\frac{ds}{dt} = 1$, $s = 0$ when $t = 0$

$$\frac{ds}{dt} = at + c_1 = 1$$

$$a(0) + c_1 = 1 \Rightarrow c_1 = 1$$

$$\frac{ds}{dt} = at + 1$$

$$s(t) = \frac{1}{2}at^2 + t + c_2 = 0$$

$$\frac{1}{2}a(0)^2 + 0 + c_2 = 0$$

$$\Rightarrow c_2 = 0$$

$$\therefore \boxed{s(t) = \frac{1}{2}at^2 + t}$$

7 $f'(x) = x^{1/2}$ $(1,1)$

$$\Rightarrow f(x) = \frac{2}{3}x^{3/2} + C$$

$$f(1) = 1 = \frac{2}{3}\cdot 1 + C \Rightarrow C = \frac{1}{3}$$

$$\therefore \boxed{f(x) = \frac{2}{3}x^{3/2} + \frac{1}{3}}$$