HW 5 – MAT 221 B

1. Prove that there is no solution to the Falkner-Skan similarity equation

$$f''' + \frac{n+1}{2} f f'' - n(f')^2 + n = 0$$

with

$$n = -1/3$$

and the boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1.$$

2. Solve numerically the Falkner-Skan similarity equation and find an approximate value of the critical power n_* at which $f''(0) = 0$ (i.e. viscous stress free).

3. This problem continues the discussion of the flow near the separation point for 2-dimensional flows. We associate the phenomena of separation to the singularity formation of the Prandtl boundary layer equation: $v(x_0, y), \partial v(x_0, y)/\partial y = \infty$ where x_0 is the separation point. From the discussion in class, the hypothesis $x_0 - x = f(y)(u - u_0)^2$ with $u_0(y) = u(x_0, y)$ then leads to the result

$$u = u_0 + \alpha(y)\sqrt{x_0 - x}, \quad v(x, y) = \frac{\beta(y)}{\sqrt{x_0 - x}}$$

where $f(y)$, $\alpha(y)$ and $\beta(y)$ are functions of y only.

- Since near the separation point v diverges and the boundary layer thickens, it is expected that in the boundary layer the two terms

$$\nu \frac{\partial^2 u}{\partial y^2}, \quad \frac{1}{\rho}\frac{dp}{dx}$$

are $O(1)$. Therefore to the leading order the Prandtl boundary layer equation reduces to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0.$$

Combine this with the continuity equation to show that v/u is independent of y and that

$$\beta(y) = Au_0(y)$$

where A is a constant.

- Use the continuity equation to show

$$\alpha(y) = 2A\frac{du_0}{dy}$$

- By the no-slip boundary condition show that at the separation point

$$u_0(0) = 0, \quad \frac{du_0}{dy}(0) = 0$$

4. Consider a free viscous layer behind a solid body in a uniform flow $(U, 0)$ as discussed in class. The boundary condition for the solution u is $u(x, \pm\infty) = U$. Derive from the boundary layer equation and the continuity equation the following conservation laws

$$\frac{d}{dx}\int_{-\infty}^{\infty} u(u-U)dy = 0$$

and

$$\frac{d}{dx}\int_{-\infty}^{\infty} (u-U)^2 dy = 0.$$