(1)

a	b	c	d	e	f	g	h	i	j
T	F	T	F	F	T	F	T	F	T

(2 pts each.)

(2.) $\det\begin{bmatrix}1&0&1\\1&1&0\\2&1&0\end{bmatrix} = \det\begin{bmatrix}1&0&0\\1&1&0\\1&0&1\end{bmatrix}(-1) = (-1)\cdot\prod_{i=1}^{3} a_{ii} = -1$

$\boxed{\text{-5 for wrong answer}}$

(3) Let $AB = BC$. [Claim $\det(A) \neq 0 \Rightarrow B = C$.] (T10)

~~proof:~~ If $\det(A) \neq 0$, then A is nonsingular, and A^{-1} exists.

So: $A^{-1}(AB) = A^{-1}(AC) \Rightarrow (A^{-1}A)B = (A^{-1}A)C$

$\Rightarrow IB = IC \Rightarrow B = C$.

| 0 pts for not showing or using $\exists A^{-1}$ |

(4) [Claim: w is orthogonal to u and v $\Rightarrow$ w is orthogonal to $ru+sv$ for all $r, s \in \mathbb{R}$.]

~~proof:~~ Let $w\cdot v = 0$ and $w\cdot u = 0$.

Consider: $w\cdot(ru+sv) = w\cdot ru + w\cdot sv = r(w\cdot u) + s(w\cdot v)$

$= r(0) + s(0) = 0$

(5) [Claim: $\|u+v\|^2 = \|u\|^2 + \|v\|^2 \Leftrightarrow u\cdot v = 0$.]

| -5 only showing ($\Rightarrow$) |

~~proof:~~ $\|u+v\|^2 = (u+v)\cdot(u+v) = u\cdot u + 2(u\cdot v) + v\cdot v$

$= \|u\|^2 + 2(u\cdot v) + \|v\|^2 \Leftrightarrow u\cdot v = 0$.

(6) $L: \mathbb{R}^2 \to \mathbb{R}^3$. $L(e_1) = \begin{bmatrix}1\\2\\-1\end{bmatrix}$, $L(e_2) = \begin{bmatrix}1\\0\\1\end{bmatrix}$

(10) (a): $A := \begin{bmatrix}1&1\\2&0\\-1&1\end{bmatrix}$. (5)

$L\left(\begin{bmatrix}2\\-1\end{bmatrix}\right) = \begin{bmatrix}1&1\\2&0\\-1&1\end{bmatrix}\cdot\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}2-1\\4\\-2-1\end{bmatrix} = \begin{bmatrix}1\\4\\-3\end{bmatrix}$

(5)

(10) (b) null space $= \{v : Av = 0\}$ want to find all v; s.t. $Av = 0$.

$\Rightarrow \begin{bmatrix}1&1\\2&0\\-1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}x+y\\2x\\-x+y\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{matrix}x+y=0\\2x=0\\-x+y=0\end{matrix}$

(5)

$\Rightarrow \begin{matrix}x=-y\\x=0\\x=y\end{matrix} \Rightarrow x = y = 0$ So $\text{null}(A) = \{0\}$.

(5)