Name: Solutions
Student ID #:

# Mini-Quiz # 10

MAT-022A-Summer Session II (8/28/09)

You have until Friday 9/4/09 by class time. Please turn in the quiz by then. You may only use a pencil (or pen) and scrap paper. Be sure to show all your work carefully. Answers with no work will not receive credit.

1. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$

   (a) Find a basis for the column space of $A$ (2.5 points).

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix} \xrightarrow[-11r_1+r_3\to r_3]{-6r_1+r_2\to r_2} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & -10 & -20 & -30 & -40 \end{bmatrix} \xrightarrow[-\frac{1}{10}r_3\to r_3]{-\frac{1}{5}r_2\to r_2} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\xrightarrow{-r_2+r_3\to r_3} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-2r_2+r_1\to r_1} \begin{bmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ RREF of A

so since column 1 & 2 in RREF A are pivots $\boxed{\left\{ \begin{bmatrix} 1 \\ 6 \\ 11 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \\ 12 \end{bmatrix} \right\} \text{ is basis for col sp}(A)}$

   (b) Find a basis for the row space of $A$ (2.5 points).

$$RREF(A) = \begin{bmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ from previous work

so $\boxed{\{[1\ 0\ -1\ -2\ -3],\ [0\ 1\ 2\ 3\ 4]\} \text{ is a basis for row sp }(A)}$

2. Find an orthonormal basis for the nullspace of $B = \begin{bmatrix} 1 & 1 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 2 & 0 & -2 & 0 & -2 \end{bmatrix}$ (5 points).

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 2 & 0 & -2 & 0 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccccc} 1 & 1 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & -2 & 2 & 0 & 2 \end{array}\right] \to \left[\begin{array}{ccccc} 1 & 1 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccccc} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} r+t \\ r+t \\ r \\ s \\ t \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

So nullspace has basis (not orthonormal) $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ with vectors $u_1$, $u_2$, $u_3$.

Let $v_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$v_2 = u_2 - \left(\frac{u_2 \cdot v_1}{v_1 \cdot v_1}\right) v_1 = u_2 - 0 = u_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$v_3 = u_3 - \left(\frac{u_3 \cdot v_1}{v_1 \cdot v_1}\right) v_1 - \left(\frac{u_3 \cdot v_2}{v_2 \cdot v_2}\right) v_2 = u_3 - \left(\frac{2}{3}\right) v_1 - 0 v_2$

$= \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 2/3 \\ 2/3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 3 \end{bmatrix}$

So $T^* = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 3 \end{bmatrix} \right\}$ is an orthogonal basis for nullspace

So $T = \left\{ \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{15}}\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 3 \end{bmatrix} \right\}$ is an orthonormal basis