Name:________________________

Student ID #:________________________

Mini-Quiz # 10

MAT-022A-Summer Session II (8/28/09)

You have until Friday 9/4/09 by class time. Please turn in the quiz by then. You may only use a pencil (or pen) and scrap paper. Be sure to show all your work carefully. Answers with no work will not receive credit.

1. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$

   (a) Find a basis for the column space of A (2.5 points).

   Solution:

   Each Column is of the form $\begin{bmatrix} 1 \\ 6 \\ 11 \end{bmatrix} \pm c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. The basis is of the following

   $$\left\{ \begin{bmatrix} 1 \\ 6 \\ 11 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

   (b) Find a basis for the row space of A (2.5 points).

   Solution:

   Each row is of the form $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix} \pm c \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$. The basis is of the following

   $$\left\{ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \right\}$$

2. Find an orthonormal basis for the nullspace of $B = \begin{bmatrix} 1 & 1 & -2 & 0 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 2 & 0 & -2 & 0 & -2 \end{bmatrix}$ (5 points).

Solution:

The RREF of the correspodning matrix is $\begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = \begin{bmatrix} c_1 - c_3 - c_5 \\ c_2 - c_3 - c_5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So we need $c_1 = c_2 = c_3 + c_5 \Rightarrow$ i.e. $\begin{bmatrix} c_3 + c_5 \\ c_3 + c_5 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$.

An normalized basis would be

$$\{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 0 \\ 0 \\ 1/\sqrt{3} \end{bmatrix} \right\}.$$

Apply Gram Schmidt process

$$\Rightarrow v_3 - < v_3, v_1 > v_1 = \begin{bmatrix} \frac{1}{3\sqrt{3}} \\ \frac{1}{3\sqrt{3}} \\ \frac{-2}{3\sqrt{3}} \\ 0 \\ 1/\sqrt{3} \end{bmatrix} := u_3$$

keep $v_1 = u_1$ and normalized u_3, the orthonormal basis is

$$\{u_1, v_2, u_3/|u_3|\} = \left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{15}} \\ \frac{1}{\sqrt{15}} \\ \frac{-2}{\sqrt{15}} \\ 0 \\ \frac{3}{\sqrt{15}} \end{bmatrix} \right\}.$$