Name:____________________
Student ID #:____________________

Mini-Quiz # 11

MAT-022A-Summer Session II (9/4/09)

You have 15 minutes. You may only use a pencil (or pen) and scrap paper.

1. Let A be a 3×3 invertible matrix which has 2 as an eigenvalue with an associated eigenvector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and -1 as an eigenvalue with an associated eigenvector $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Now compute the following. (2 points each)

   (a) $A^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   Solution:

   $$A^{-1}v = \lambda^{-1}v \text{ if } Av = \lambda v \Rightarrow A^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1/2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

   (b) $A \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

   Solution:

   $$A \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} = A(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}) = A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - A \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

(c) Compute the eigenvalues and basis for the associated eigenspaces for the following matrix (6 points)

$$\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

Solution:

$$\det \lambda I - A = \det \begin{bmatrix} \lambda - 3 & 0 & 0 \\ -1 & \lambda - 2 & -1 \\ 0 & 0 & \lambda - 3 \end{bmatrix} = (\lambda - 3)^2(\lambda - 3) = 0$$

- $\lambda = 3$

  $$(3I - A)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x = y - z.$$

  So eigenvectors are $\begin{bmatrix} y - z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

  The basis is $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

- $\lambda = 2$ $(2I - A) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. The RREF is of the form $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Clearly the basis is

  $$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$