Name:_____
Student ID #:_____

Mini-Quiz # 12

MAT-022A-Summer Session II (9/6/09)

You have until the start of class on Wednesday 9/9/09 to finish. You may only use a pencil (or pen) and scrap paper.

1. Is the following matrix A diagonalizable and if so find invertible P so that $P^{-1}AP$ is diagonal. (5 points)

$$A = \begin{bmatrix} 3 & 0 & 1 \\ -1 & 2 & -1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution:

The characteristic polynomial is given by $\det(\lambda I - A) = \lambda^3 - 8\lambda^2 + 20\lambda - 16 = (\lambda-2)^2(\lambda-4)$.

- $\lambda = 2$:

  $2I - A = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$. The RREF is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Thus the eigenvectors are$(0, 1, 0)$ and $(-1, 0, 1)$.

- $\lambda = 4$**:**

  $4I - A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$. The RREF is $\begin{bmatrix} 1 & 0 & -1 \\ 010 & & \\ 0 & 0 & 0 \end{bmatrix}$, thus the eigenvector is $(1, -1, 1)$.

The matrix given is diagonalizable because we have three independent eigenvector and

$$P = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}.$$

2. Determine whether or not the following matrix is diagonalizable. If it is diagonalizable find an invertible P so that $P^{-1}AP$ is diagonal. (5 points)

$$A = \begin{bmatrix} 3 & 1 & 1 & -1 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution:
The characteristic polynomial is given by $\det(\lambda I - A) = (\lambda - 3)^3\lambda$.

When $\lambda = 3$, $3I - A = \begin{bmatrix} 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$.

The RREF of the matrix is $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, so clearly the eigenvector is $(1, 0, 0, 0)$.

Up to this point we can see the matrix is not diagonalizable since the dimension of the eigenspace can be at most 2.