Name: Solutions

Student ID #:

Mini-Quiz # 2

MAT-022A-Summer Session II (8/5/09)

You have 10 minutes. You may only use a pencil (or pen) and scrap paper. No calculators, notes or books.

1.

(a) Give a definition of a linear combination of the vectors $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix}$. (Use words not just symbols. 4 points.)

It is any possible sum $x_1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix}$

where x_1, x_2 can be any real numbers.

(b) Is the vector $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ a linear combo of the vectors $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$? (You must give a careful justification of your answer to receive credit. Explain with words. You might want to use the reverse side. 6 points.)

An equivalent question is can we find values for x_1, x_2 so that the equation

$$x_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$ has a solution?

This will yield the system
$$\begin{aligned} x_2 &= 2 \\ x_1 + x_2 &= 1 \\ x_1 &= 1 \end{aligned}$$

which in augmented form is $\left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right]$

(back)

Now lets simplify

$$\left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{-r_1+r_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-r_3+r_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 0 & -2 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow[\text{system form}]{\text{in ordinary}} \begin{array}{l} x_1 + 0x_2 = 2 \\ 0x_1 + 0x_2 = -2 \\ 0x_1 + x_2 = 1 \end{array}$$

but equation 2 in this system says $0 = -2$ which is never true, so there is no solution.
(Note that we used elementary transformations to change to ~~an~~ a different but equivalent system)

Thus $x_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ has no solution for the unknowns x_1, x_2 which means that $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is <u>not</u> a linear combination of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.