Name:____________________________

Student ID #:____________________________

Mini-Quiz # 4

MAT-022A-Summer Session II (8/7/09)

You have 10 minutes. You may only use a pencil (or pen) and scrap paper. No calculators, notes or books.

1. For the following proposed questions write a linear system in augmented form which could be solved to answer the question. You do not have to solve the system and you do not need to explain your answer. (2 points each).

   (a) For $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 5 & 1 & 4 \end{bmatrix}$ let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation whose image $f(u) = A^T u$ for all $u \in \mathbb{R}^3$. Is $\begin{bmatrix} 1 \\ -4 \\ \pi^2 \end{bmatrix}$ in the range of f?

   Solution:

   $$A^T = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

   Let $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$, we have to solve the following.

   $$f(u) = A^t u = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ \pi^2 \end{bmatrix}.$$

   $$\begin{bmatrix} 1 & 0 & 5 & | & 1 \\ 2 & 0 & 1 & | & -4 \\ 0 & 0 & 4 & | & \pi^2 \end{bmatrix}$$

   (b) Is $\begin{bmatrix} 4 \\ 7 \\ 2 \\ 1 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 3 \\ 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$, $\begin{bmatrix} \pi \\ 7 \\ -2 \\ \frac{1}{2} \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$?

   Solution:

   So we need to see if the following equantions has a solution (x, y, z).

   $$\begin{bmatrix} 4 \\ 7 \\ 2 \\ 1 \end{bmatrix} = x \begin{bmatrix} 3 \\ 1 \\ \sqrt{2} \\ 1 \end{bmatrix} + y \begin{bmatrix} \pi \\ 7 \\ -2 \\ \frac{1}{2} \end{bmatrix} + z \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

i.e.

$$\begin{bmatrix} 3 & \pi & 0 & | & 4 \\ 1 & 7 & 1 & | & 7 \\ \sqrt{2} & -2 & 1 & | & 2 \\ 1 & 1/2 & 0 & | & 1 \end{bmatrix}$$

2. Carefully using algebraic properties show the following. (3 points each)

(a) Let A be an $n \times n$ matrix. Show that AA^T is a symmetric matrix.
Solution:
Using the rules for matrices operations and transpose, $(AB)^t = B^t A^t$ and $(A^t)^t = A^t$, we have $(AA^t)^t = (A^t)^t A^t = AA^t$. So AA^t is symmetric.

(b) Let x_h be a solution to the homogeneous system $Ax = 0$ and let x_p be a solution to the linear system $Ax = b$. Show that $x_h + x_p$ is a solution to the system $Ax = b$.
Solution:
We know $Ax_h = 0$ and $Ax_p = b$. Thus,by the linear property of the matrix operator A

$$A(x_h + x_p) = A(x_h) + A(x_p) = 0 + b = b.$$

Therefore $x_h + x_p$ is a solution to the system $Ax = b$.