Name:____________________
Student ID #:____________________

Mini-Quiz # 5

MAT-022A-Summer Session II (8/12/09)

You have 15 minutes. You may only use a pencil (or pen) and scrap paper. You must show all your work carefully. No calculators, notes or books.

1. For the following matrices compute their inverses if possible or state otherwise. (3 points each).

(a) $A = \begin{bmatrix} 2 & 5 \\ 1 & -3 \end{bmatrix}$

$$\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & -3 & 0 & 1 \end{array}\right) \xrightarrow{-\frac{1}{2}R_1+R_2 \to R_2} \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & -11/2 & -1/2 & 1 \end{array}\right)$$

$$\xrightarrow{-2/11 R_2 \to R_2} \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & 1 & 1/11 & -2/11 \end{array}\right) \xrightarrow{R_1 - 5R_2 \to R_1} \left(\begin{array}{cc|cc} 2 & 0 & 6/11 & 10/11 \\ 0 & 1 & 1/11 & -2/11 \end{array}\right) \xrightarrow{1/2 R_1 \to R_1} \left(\begin{array}{cc|cc} 1 & 0 & 3/11 & 5/11 \\ 0 & 1 & 1/11 & -2/11 \end{array}\right)$$

$$\Rightarrow \boxed{A^{-1} = \begin{pmatrix} 3/11 & 5/11 \\ 1/11 & -2/11 \end{pmatrix}} \text{ or } \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix} \cdot 1/11$$

(b) $B = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 5 \\ 1 & 1 & -1 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 1 & -1 & 5 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-R_1+R_2 \to R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-R_1+R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 1 & 0 \\ 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right) \xrightarrow{R_2+R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \boxed{B \text{ has no inverse}}$$

2. Compute the following determinant. (4 points)

$$\det \begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix}$$

$$A = \begin{pmatrix} 2 & 2 & 0 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{pmatrix} \xrightarrow[R_2 - R_1 \to R_2]{(1)} \begin{pmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 4 \end{pmatrix} \xrightarrow[2R_3 - R_1 \to R_3]{(2)} \begin{pmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 2 & 0 & 0 & 4 \end{pmatrix}$$

$$\xrightarrow[R_4 - R_1 \to R_4]{(1)} \begin{pmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & -2 & 0 & 3 \end{pmatrix} \xrightarrow[\text{Swap rows } R_3 \text{ \& } R_4]{(-1)^{(1)}} \begin{pmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix} \xrightarrow[R_3 + 2R_2 \to R_3]{(1)} \begin{pmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix} = U$$

$$(1)\cdot(2)\cdot(1)\cdot(-1)^1\cdot(1)\cdot \det A = \det U$$

$$\Rightarrow \det A = \frac{-1}{2} \det U$$

$$\det U = \det \begin{bmatrix} 2 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} = -4$$

$$\det A = \frac{-1}{2}(-4) = \boxed{2 = \det A}$$