Name:______________________________
Student ID #:______________________________

Mini-Quiz # 7

MAT-022A-Summer Session II (8/21/09)

You have until Monday 8/24/09 by class time. You may only use a pencil (or pen) and scrap paper. You must show all your work carefully.

1. Let $V = M_{22}(\mathbb{R})$ = set of 2×2 matrices with the following operations:
   For any $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ in V and any r in $\mathbb{R}$ define
   $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \oplus \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} a+x & by \\ cz & d+w \end{bmatrix}$ and $r \odot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ra & rb \\ rc & rd \end{bmatrix}$

   (a) Prove or disprove:
   The matrix $U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the $\oplus$-identity (i.e. for all A in V we have $A \oplus U = U \oplus A = A$). (3 points)

   Solution:

   True.

   $$A \oplus U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0+a & 1 \cdot b \\ 1 \cdot c & 0+d \end{bmatrix}$$

   $$U \oplus A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+0 & b \cdot 1 \\ c \cdot 1 & d+0 \end{bmatrix}$$

   Both equal to $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

   (b) Prove or disprove: For all scalars r, s in $\mathbb{R}$ and all A in V we have: $(r+s) \odot A = r \odot A \oplus s \odot A$.
   (3 points)
   Solution: False.

   $$(r+s) \odot A = \begin{bmatrix} (r+s)a & (r+s)b \\ (r+s)c & (r+s)d \end{bmatrix} \neq r \odot A \oplus s \odot A = \begin{bmatrix} (r+s)a & (rs)b^2 \\ (rs)c^2 & (r+s)d \end{bmatrix}$$

   (c) Now circle all of the vector space axioms below which are satisfied by the set M_{22} with the above defined operations $\oplus, \odot$. They are listed using the books vector space axiom notation as seen on page 272 (I also used this notation in class). You do not need to worry about the closure axioms α and β which are satisfied. You do not need to include work on this problem (but you should work out the answers for yourself as practice). (4 points)

(a), (b), (c), (d), (e), (f), (g), (h)

Answers:

True:a,b,c,g,h.

Remark: notice the matrx $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$ has no inverse.