Name:____________________________
Student ID #:____________________________

Mini-Quiz # 8

MAT-022A-Summer Session II (8/26/09)

You have 15 minutes. You may only use a pencil (or pen) and scrap paper. You must show all your work carefully.

1. Do the matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, and $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ span $V = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid \text{where } a, b, c \text{ are any real numbers} \right\}$ = the vector space of all 2×2 upper triangular matrices? (5 points)

Solution:
Given an upper triangular matrix $\begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$, if we want to express it as the linear combination of the given three matrices, we must have the following:

$$a \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + c \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a+b-c & a+2b \\ 0 & a+3b+c \end{bmatrix} = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$

i.e.

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 & | & a \\ 1 & 2 & 0 & | & b \\ 1 & 3 & 1 & | & c \end{bmatrix}.$$

Putting the above in reduced echelon form, we have the following

$$\begin{bmatrix} 0 & 1 & 1 & | & b-a \\ 1 & 2 & 0 & | & b \\ 0 & 0 & 0 & | & c+a-2b \end{bmatrix} = 0.$$

It is impossible to have $0 = c + a - 2b$ for arbitrary (a, b, c), therefore the given three matrices do not span V.

2. Are $t^3 - t + 1, \quad t^3 + t - 1, \quad 2t^3$ linearly independent in P_3? (5 points)

Solution:
Notice the linear combination of the three polnomials equal to

$$a(t^3 - t + 1) + b(t^3 + t - 1) + c(2t^3).$$

if they are linearly independent, we must have

$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow (a, b, c) = (0, 0, 0)$$

But this is impossible since the row reduced echelon form of the system $\begin{bmatrix} 1 & 1 & 2 & | & 0 \\ -1 & 1 & 0 & | & 0 \\ 1 & -1 & 0 & | & 0 \end{bmatrix}$ is

$$\begin{bmatrix} 1 & 0 & 1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

which has non trivial solution,so they are not linearly independent.