Math 108 Fall 2026

HW 3 SOLUTIONS

2.1 5(d) True, 5(j) False.

6(d) $A = B = \{1\}$, $C = \{2\}$.

14(d) $\mathcal{P}(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$, then substitute $a=1$, $b=\{\emptyset\}$, $c=\{2,\{3\}\}$.

15(c) True. If $x \in A$, then $\{x\} \subseteq A$, so $\{x\} \in \mathcal{P}(A)$.

15(h) True. If $B \in \mathcal{P}(A)$, then $B \subseteq A$; as $C \in B$, $C \in A$, so $C \in \mathcal{P}(A)$.

17(d) True. $\emptyset$ is a subset of any set.

19(g) F. Completely wrong. The statement is not true, e.g. $\mathcal{P}(\{1\}) = \{\emptyset, \{1\}\}$, but $\{1\} \notin \{\emptyset, \{1\}\}$ as $1 \notin \{\emptyset, \{1\}\}$. What is true is that $A \in \mathcal{P}(A)$. The mistake in the proof is confusion between "$\in$" and "$\subseteq$".

2.2 1(j) $A \cup B = \{1,2,3,4,5,6,7,8,9\}$, $C \cap D = \{1,2,5,7,8\}$, so $(A \cup B) - (C \cap D) = \{3,4,6,9\}$.

2(f) $[2,5]$.

9(b) Assume $x \in A$. Then $x \in B \cup C$, so either $x \in B$ or $x \in C$. But $x \in B$ implies $x \notin A$ as $A \cap B = \emptyset$. So we must have $x \in C$.

9(c) $(A-B)-C = (A \cap B^c) \cap C^c$

$(A-C)-(B-C) = (A \cap C^c) \cap (B \cap C^c)^c = A \cap (C^c \cap (B^c \cup C))$

$= A \cap ((C^c \cap B^c) \cup \underbrace{(C^c \cap C)}_{=\emptyset}) = A \cap (C^c \cap B^c)$

11(b) $A = \{1\}$ $B = C = \emptyset$.

12(b) If $X \in \mathcal{P}(A) \cup \mathcal{P}(B)$, then $X \subseteq A$ or $X \subseteq B$. In either case, $X \subseteq A \cup B$, so $X \in \mathcal{P}(A \cup B)$.

12(c) $A = \{1\}$, $B = \{2\}$, $A \cup B = \{1,2\}$; $A \cup B \notin \mathcal{P}(A)$, $A \cup B \notin \mathcal{P}(B)$, so $A \cup B \notin \mathcal{P}(A) \cup \mathcal{P}(B)$. The equality holds if and only if $A \subseteq B$ or $B \subseteq A$.

HW 3 solutions, cont'd

19(f) F. The claim is not true, e.g. take $A=\{1,2\}$, $B=\{2,3\}$, $C=\{3,4\}$. The mistake in the proof is that "the two x's are not the same necessarily," i.e. there exist $x_1 \in A \cap B$ and $x_2 \in B \cap C$.

2.3 1(n) $\bigcap_{n\in\mathbb{N}} D_n = (-1, 0]$, $\bigcup_{n\in\mathbb{N}} D_n = (-\infty, 1)$.