Math 108, Fall 2026

HW 4 SOLUTIONS

2.4 6g $(n=1)$ $\frac{1}{1\cdot 2} = \frac{1}{1+1}$.

$(n \to n+1)$ $\frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} \overset{\text{by the I.H.}}{=} \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)}$

$= \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2}$

7a $(n=1)$ $1+5+6 = 12 = 4\cdot 3$,

$(n \to n+1)$ $(n+1)^3 + 5(n+1) + 6 = n^3 + 3n^2 + 3n + 1 + 5n + 5 + 6$

$= n^3 + 5n + 6 + 3(n^2 + n + 2) \overset{\text{I.H.}}{=} 3k + 3(n^2+n+2)$,

for some $k \in \mathbb{Z}$.

7h $(n=1)$ $3 \geq 1+2$

$(n \to n+1)$ $3^{n+1} \geq 3 \cdot 3^n \overset{\text{I.H.}}{\geq} 3(1+2^n) = 3 + 3\cdot 2^n \geq 1 + 2\cdot 2^n$

$= 1 + 2^{n+1}$.

8h $(n=2)$ $\sqrt{2} < 1 + \frac{1}{\sqrt{2}}$ as $2 < \sqrt{2}+1$.

$(n \to n+1)$ $\sqrt{n+1} = \sqrt{n} + \sqrt{n+1} - \sqrt{n} = \sqrt{n} + \frac{1}{\sqrt{n+1} + \sqrt{n}}$

$< \sqrt{n} + \frac{1}{\sqrt{n+1}} \overset{\text{I.H.}}{\leq} \frac{1}{\sqrt{1}} + \cdots + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}}$

13(c) F. $n^2 + n = n(n+1)$ is always even. Verification for $n=1$ was wrong, although the inductive step is correct.

Math 108; Fall 2026

HW4 SOLUTIONS, CONT'D

2.5

1(b) Check for 34, 35, 36, 37, 38. Now take $n \geq 39$.
By I.H., $n-5$ can be written as stated: $n-5 = 4s + 5t$.
Then $n = 4s + 5(t+1)$. □

2. $(n=1, n=2)$ True.
$(n \geq 3)(n-1 \to n)$ $a_n = 5a_{n-1} - 6a_{n-2}$, so, using IH,
$a_n = 5 \cdot 2^{n-1} - 6 \cdot 2^{n-2} = 10 \cdot 2^{n-2} - 6 \cdot 2^{n-2} = 4 \cdot 2^{n-2} = 2^n$. □

5(b). $(n=1, n=2)$ True, as $f_1 = f_2 = 1$.
$(n \geq 3)(n-1 \to n)$ We will show that $\gcd(f_n, f_{n-1}) = 1$ assuming that $\gcd(f_{n-1}, f_{n-2}) = 1$. If $d \mid f_n$ and $d \mid f_{n-1}$, then $d \mid (f_n - f_{n-1})$, but $f_n - f_{n-1} = f_{n-2}$, so $d \mid f_{n-2}$ and $d = 1$ by IH.

5(c). If $d \mid f_{n+2}$, $d \mid f_n$, then $d \mid f_{n+2} - f_n$, $d \mid f_{n+1}$. So $d = 1$ by 5(b).

13(c) The statement is true, but the proof is badly written. The base case is not checked, and the use of induction hypothesis is unclear.