Math 108, Fall 2026.

Modular arithmetic problems

1. All digits in these problems are decimal digits. Let A be the sum of digits of the number 4444^{4444} and B the sum of digits of A. Find the sum of digits of B.

2. What are the last two digits of 7^{7^7} ?

3. For which $n \in \mathbb{N}$ is $2^n + 1$ divisible by 7? How about $2^n - 1$?

4. On a Sunday, somebody asks you: "What day of the week will be 10^{100} days from today (assuming tomorrow, Monday, is one day from today)?" Answer this (highly hypothetical) question.

Solutions

1. All digits in these problems are decimal digits. Let A be the sum of digits of the number 4444^{4444} and B the sum of digits of A. Find the sum of digits of B.

Solution. Denote by C the sum of digits of B. Then $C \geq 1$, and we will first find an upper bound for C. As $4444^{4444} < 10^{4 \cdot 10^4}$, $A \leq 9 \cdot 4 \cdot 10^4 < 10^6$, and then $B \leq 9 \cdot 6 = 54$. Therefore $C \leq 4 + 9 = 13$.

Now we compute $C \mod 9$. As $4444 = 16 = 7 = -2 \mod 9$, $4444^3 = -8 = 1 \mod 9$. Moreover, $4444 = 16 = 1 \mod 3$, and so

$$4444^{4444} = 7^{4444} = 7 \mod 9.$$

It follows that also $C = 7 \mod 9$, but we also know that $1 \leq C \leq 13$, so $C = 7$.

2. What are the last two digits of $7^{7^{7^7}}$?

Solution. Let N be this (enormous) number. We are asked to find $N \mod 100$. We start by looking at powers of 7 modulo 100:

n	$7^n \mod 100$
1	7
2	49
3	$7 \cdot 49 = 343 = 43$
4	$7 \cdot 43 = 301 = 1$

Therefore, we need to compute $7^{7^7} \mod 4$, which necessitates looking at powers of 7 modulo 4, but $7^n = (-1)^n \mod 4$, so odd powers are $-1 \mod 4$ and even powers are $1 \mod 4$. As 7^7 is odd, $7^{7^7} = -1 = 3 \mod 4$. It now follows that

$$N = 7^3 = 43 \mod 100,$$

so the answer is 43.

3. For which $n \in \mathbb{N}$ is $2^n + 1$ divisible by 7? How about $2^n - 1$?

Solution. We start by looking at powers of 2 modulo 7:

n	$2^n \mod 7$
1	2
2	4
3	1

and now the $2 - 4 - 1$ pattern repeats. It follows that $2^n + 1$ is never divisible by 7, as the only possible remainders are 3, 5, and 2. On the other hand, $2^n - 1$ is divisible by 7 exactly when $3 \mid n$ (and the other remainders are 1 and 3).

4. On a Sunday, somebody asks you: "What day of the week will be 10^{100} days from today (assuming tomorrow, Monday, is one day from today)?" Answer this (highly hypothetical) question.

Solution. We need to compute $10^{100} = 3^{100} \mod 7$. We look at powers of 3 modulo 7:

n	$3^n \mod 7$
1	3
2	2
3	-1
4	-3
5	-2
6	1

As $3^6 = 1 \mod 7$ and $100 = 16 = 4 \mod 6$, we conclude that $3^{100} = 3^4 = -3 = 4 \mod 7$. That day will be Thursday.