

### Homework 5

1. A group of 100 people is tested for a disease, for which there is an infallible but expensive blood test. They are divided into 10 groups of 10 people each, then the people in each group pool their blood. If a test for a group comes out negative, everybody in that group is healthy and no more tests are done. Otherwise, the blood of each of the 10 people in the group is tested separately. Assume that the probability that a person has the disease is, independently, 0.1. Compute the expected number of tests performed.
2. A bag contains 5 red and 5 blue balls. Select two at random without replacement. If they are of the same color you win \$1.10 otherwise you lose \$1. Let  $X$  be your winnings. Compute  $EX$  and  $\text{Var}(X)$ .
3. A multiple choice exam has 5 questions, with three choices for each question. The passing score is at least four correct answers. (a) What is the probability that a student who answers each question at random passes the test? (b) Assuming a class has 50 students, and all answer each question at random, what is the expected number of students that pass the test?
4. Assume the suicide rate is 1 per 100,000 people per month, and a city has 400,000 inhabitants. (a) Find the probability that there will be 8 or more suicides next month in this city. (b) Find the probability that next year there will be at least two months with 8 or more suicides. (c) Counting the next month as month 1, what is the probability that the first month to have 8 or more suicides will be month  $i$ ?

*You should also do problems 1, 2, 4, 5 in Section 5 of the book.*

## Solutions

1. Each group independently has either 1 or 11 tests. The probability that it has 1 test is  $0.9^{10}$ . The number  $N$  of groups that have 1 test thus is  $\text{Binomial}(10, 0.9^{10})$ . The number of tests is  $11 \cdot (10 - N) + N = 110 - 10N$ , with the expectation  $110 - 10 \cdot 10 \cdot 0.9^{10} \approx 75.1$ .
2. Let  $I$  be the indicator of the event that you win. Then  $X = 2.1I - 1$ . The probability  $p$  of winning is  $20/45$ , so  $EX = -3/45 = -1/15$ , and  $\text{Var}(X) = (2.1)^2\text{Var}(I) = (2.1)^2(20 \cdot 25)/45^2 = 49/45$ .
3. (a) The number of correct answers is  $\text{Binomial}(5, 1/3)$ , so the answer is  $p = \binom{5}{4} \left(\frac{1}{3}\right)^4 \frac{2}{3} + \binom{5}{5} \left(\frac{1}{3}\right)^5 \approx 0.04527$ . (b) The number of students who pass is  $\text{Binomial}(50, p)$ , so the answer is  $50p \approx 0.2263$ .
4. (a) Approximate with Poisson with  $\lambda = 4$ , to get  $p = 1 - \sum_{i=0}^7 \frac{\lambda^i}{i!} e^{-\lambda}$ . (b) Then the number of months with 8 or more suicides is  $\text{Binomial}(12, p)$ , so the answer is  $1 - (1 - p)^{12} - 12p(1 - p)^{11}$ , and (c)  $(1 - p)^{i-1}p$ .