43. ***Retail Price*** The average retail price P (in dollars) of 1 pound of 100% ground beef from 1996 to 2002 can be modeled by the equation

$$P = -0.001059t^4 + 0.03015t^3 - 0.2850t^2 + 1.007t + 0.50$$

where t is the year, with $t = 6$ corresponding to 1996. *(Source: U.S. Bureau of Labor Statistics)*

(a) Find the rate of change of the price with respect to the year.

(b) At what rate was the price of 100% ground beef changing in 1997? in 2000? in 2002?

(c) Use a graphing utility to graph the function for $6 \le t \le 12$. During which years was the price increasing? decreasing?

(d) For what years do the slopes of the tangent lines appear to be positive? negative?

(e) Compare your answers for parts (c) and (d).

44. ***Recycling*** The amount T of recycled paper products in millions of tons from 1993 to 2001 can be modeled by the equation

$$T = \sqrt{2.4890t^3 - 62.062t^2 + 553.16t - 509.4}$$

where t is the year, with $t = 3$ corresponding to 1993. *(Source: Franklin Associates, Ltd.)*

(a) Use a graphing utility to graph the equation. Be sure to choose an appropriate window.

(b) Determine dT/dt. Evaluate dT/dt for 1993, 1998, and 2001.

(c) Is dT/dt positive for $t \ge 3$? Does this agree with the graph of the function? What does this tell you about this situation? Explain your reasoning.

45. ***Velocity*** A rock is dropped from a tower on the Brooklyn Bridge, 276 feet above the East River. Let t represent the time in seconds.

(a) Write a model for the position function (assume that air resistance is negligible).

(b) Find the average velocity during the first 2 seconds.

(c) Find the instantaneous velocities when $t = 2$ and $t = 3$.

(d) How long will it take for the rock to hit the water?

(e) When it hits the water, what is the rock's speed?

46. ***Velocity*** The straight-line distance s (in feet) traveled by an accelerating bicyclist can be modeled by

$$s = 2t^{3/2}, \quad 0 \le t \le 8$$

where t is the time (in seconds). Complete the table, showing the velocity of the bicyclist at two-second intervals.

Table for 46

Time, t	0	2	4	6	8
Velocity					

47. ***Cost, Revenue, and Profit*** The fixed cost of operating a small flower shop is \$2500 per month. The average cost of a floral arrangement is \$15 and the average price is \$27.50. Write the monthly revenue, cost, and profit functions for the floral shop in terms of x, the number of arrangements sold.

48. ***Profit*** The weekly demand and cost functions for a product are given by

$$p = 1.89 - 0.0083x \quad \text{and} \quad C = 21 + 0.65x.$$

Write the profit function for this product.

Marginal Cost In Exercises 49–52, find the marginal cost function.

49. $C = 2500 + 320x$

50. $C = 225x + 4500$

51. $C = 370 + 2.55\sqrt{x}$

52. $C = 475 + 5.25x^{2/3}$

Marginal Revenue In Exercises 53–56, find the marginal revenue function.

53. $R = 200x - \frac{1}{5}x^2$

54. $R = 150x - \frac{3}{4}x^2$

55. $R = \dfrac{35x}{\sqrt{x-2}}, \quad x \ge 6$

56. $R = x\left(5 + \dfrac{10}{\sqrt{x}}\right)$

Marginal Profit In Exercises 57 and 58, find the marginal profit function.

57. $P = -0.0002x^3 + 6x^2 - x - 2000$

58. $P = -\frac{1}{15}x^3 + 4000x^2 - 120x - 144{,}000$

In Exercises 59–78, find the derivative of the function. Simplify your result.

59. $f(x) = x^3(5 - 3x^2)$

60. $y = (3x^2 + 7)(x^2 - 2x)$

61. $y = (4x - 3)(x^3 - 2x^2)$

62. $s = \left(4 - \dfrac{1}{t^2}\right)(t^2 - 3t)$

63. $f(x) = \dfrac{6x - 5}{x^2 + 1}$

64. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

65. $f(x) = (5x^2 + 2)^3$

66. $f(x) = \sqrt[3]{x^2 - 1}$

67. $h(x) = \dfrac{2}{\sqrt{x + 1}}$

68. $g(x) = \sqrt{x^6 - 12x^3 + 9}$

69. $g(x) = x\sqrt{x^2 + 1}$

70. $g(t) = \dfrac{t}{(1 - t)^3}$

71. $f(x) = x(1 - 4x^2)^2$

72. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5$

73. $h(x) = [x^2(2x + 3)]^3$

74. $f(x) = [(x - 2)(x + 4)]^2$

75. $f(x) = x^2(x - 1)^5$

76. $f(s) = s^3(s^2 - 1)^{5/2}$

77. $h(t) = \dfrac{\sqrt{3t + 1}}{(1 - 3t)^2}$

78. $g(x) = \dfrac{(3x + 1)^2}{(x^2 + 1)^2}$

79. ***Physical Science*** The temperature T (in degrees Fahrenheit) of food placed in a freezer can be modeled by

$$T = \frac{1300}{t^2 + 2t + 25}$$

where t is the time (in hours).

(a) Find the rates of change of T when $t = 1$, $t = 3$, $t = 5$, and $t = 10$.

(b) Graph the model on a graphing utility and describe the rate at which the temperature is changing.

80. ***Forestry*** According to the *Doyle Log Rule*, the volume V (in board-feet) of a log of length L (feet) and diameter D (inches) at the small end is

$$V = \left(\frac{D - 4}{4}\right)^2 L.$$

Find the rates at which the volume is changing with respect to D for a 12-foot-long log whose smallest diameter is (a) 8 inches, (b) 16 inches, (c) 24 inches, and (d) 36 inches.

In Exercises 81–88, find the given derivative.

81. Given $f(x) = 3x^2 + 7x + 1$, find $f''(x)$.

82. Given $f'(x) = 5x^4 - 6x^2 + 2x$, find $f'''(x)$.

83. Given $f'''(x) = -\dfrac{6}{x^4}$, find $f^{(5)}(x)$.

84. Given $f(x) = \sqrt{x}$, find $f^{(4)}(x)$.

85. Given $f'(x) = 7x^{5/2}$, find $f''(x)$.

86. Given $f(x) = x^2 + \dfrac{3}{x}$, find $f''(x)$.

87. Given $f''(x) = 6\sqrt[3]{x}$, find $f'''(x)$.

88. Given $f'''(x) = 20x^4 - \dfrac{2}{x^3}$, find $f^{(5)}(x)$.

89. ***Athletics*** A person dives from a 30-foot platform with an initial velocity of 5 feet per second (upward).

(a) Find the position function of the diver.

(b) How long will it take for the diver to hit the water?

(c) What is the diver's velocity at impact?

(d) What is the diver's acceleration at impact?

90. ***Velocity and Acceleration*** The position function of a particle is given by

$$s = \frac{1}{t^2 + 2t + 1}$$

where s is the height (in feet) and t is the time (in seconds). Find the velocity and acceleration functions.

In Exercises 91–94, use implicit differentiation to find dy/dx.

91. $x^2 + 3xy + y^3 = 10$

92. $x^2 + 9xy + y^2 = 0$

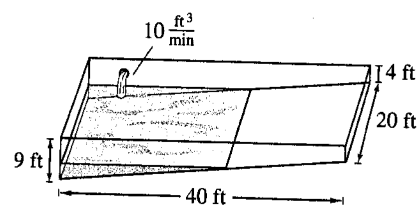
93. $y^2 - x^2 + 8x - 9y - 1 = 0$

94. $y^2 + x^2 - 6y - 2x - 5 = 0$

In Exercises 95–98, use implicit differentiation to find an equation of the tangent line at the given point.

	Equation	*Point*
95.	$y^2 = x - y$	$(2, 1)$
96.	$2\sqrt[3]{x} + 3\sqrt{y} = 10$	$(8, 4)$
97.	$y^2 - 2x = xy$	$(1, 2)$
98.	$y^3 - 2x^2y + 3xy^2 = -1$	$(0, -1)$

99. ***Water Level*** A swimming pool is 40 feet long, 20 feet wide, 4 feet deep at the shallow end, and 9 feet deep at the deep end (see figure). Water is being pumped into the pool at the rate of 10 cubic feet per minute. How fast is the water level rising when there is 4 feet of water in the deep end?



100. ***Profit*** The demand and cost functions for a product can be modeled by

$$p = 211 - 0.002x$$

and

$$C = 30x + 1{,}500{,}000$$

where x is the number of units produced.

(a) Write the profit function for this product.

(b) Find the marginal profit when 80,000 units are produced.

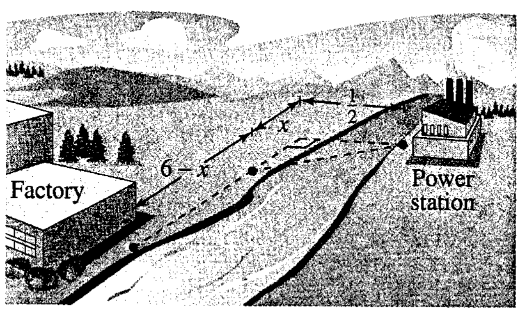
(c) Graph the profit function on a graphing utility and use the graph to determine the price you would charge for the product. Explain your reasoning.

20. ***Maximum Profit*** A real estate office handles a 50-unit apartment complex. When the rent is \$580 per month, all units are occupied. For each \$40 increase in rent, however, an average of one unit becomes vacant. Each occupied unit requires an average of \$45 per month for service and repairs. What rent should be charged to obtain a maximum profit?

21. ***Maximum Revenue*** When a wholesaler sold a product at \$40 per unit, sales were 300 units per week. After a price increase of \$5, however, the average number of units sold dropped to 275 per week. Assuming that the demand function is linear, what price per unit will yield a maximum total revenue?

22. ***Maximum Profit*** Assume that the amount of money deposited in a bank is proportional to the square of the interest rate the bank pays on the money. Furthermore, the bank can reinvest the money at 12% simple interest. Find the interest rate the bank should pay to maximize its profit.

23. ***Minimum Cost*** A power station is on one side of a river that is 0.5 mile wide, and a factory is 6 miles downstream on the other side of the river (see figure). It costs \$6 per foot to run overland power lines and \$8 per foot to run underwater power lines. Write a cost function for running the power lines from the power station to the factory. Use a graphing utility to graph your function. Estimate the value of x that minimizes the cost. Explain your results.



24. ***Minimum Cost*** An offshore oil well is 1 mile off the coast. The oil refinery is 2 miles down the coast. Laying pipe in the ocean is twice as expensive as laying it on land. Find the most economical path for the pipe from the well to the oil refinery.

25. ***Minimum Cost*** A small business uses a minivan to make deliveries. The cost per hour for fuel is $C = v^2/600$, where v is the speed of the minivan (in miles per hour). The driver is paid \$10 per hour. Find the speed that minimizes the cost of a 110-mile trip. (Assume there are no costs other than fuel and wages.)

26. ***Minimum Cost*** Repeat Exercise 25 for a fuel cost per hour of

$$C = \frac{v^2 + 360}{720}$$

and a wage of \$8 per hour.

Elasticity In Exercises 27–32, find the price elasticity of demand for the demand function at the indicated x-value. Is the demand elastic, inelastic, or of unit elasticity at the indicated x-value? Use a graphing utility to graph the revenue function, and identify the intervals of elasticity and inelasticity.

	Demand Function	*Quantity Demanded*
27.	$p = 400 - 3x$	$x = 20$
28.	$p = 5 - 0.03x$	$x = 100$
29.	$p = 20 - 0.0002x$	$x = 30$
30.	$p = \dfrac{500}{x + 2}$	$x = 23$
31.	$p = \dfrac{100}{x^2} + 2$	$x = 10$
32.	$p = 100 - \sqrt{0.2x}$	$x = 125$

33. ***Elasticity*** The demand function for a product is given by

$x = p^2 - 20p + 100$.

(a) Consider a price of \$2. If the price increases by 5%, what is the corresponding percent change in the quantity demanded?

(b) Average elasticity of demand is defined to be the percent change in quantity divided by the percent change in price. Use the percent in part (a) to find the average elasticity over the interval $[2, 2.1]$.

(c) Find the elasticity for a price of \$2 and compare the result with that in part (b).

(d) Find an expression for the total revenue and find the values of x and p that maximize the total revenue.

34. ***Elasticity*** The demand function for a product is given by

$p^3 + x^3 = 9$.

(a) Find the price elasticity of demand when $x = 2$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

35. ***Elasticity*** The demand function for a product is given by

$p = 20 - 0.02x, \quad 0 < x < 1000$.

(a) Find the price elasticity of demand when $x = 560$.

(b) Find the values of x and p that maximize the total revenue.

(c) For the value of x found in part (b), show that the price elasticity of demand has unit elasticity.

3 CHAPTER REVIEW EXERCISES

In Exercises 1–4, find the critical numbers of the function.

1. $f(x) = -x^2 + 2x + 4$

2. $g(x) = (x - 1)^2(x - 3)$

3. $h(x) = \sqrt{x}(x - 3)$

4. $f(x) = (x + 1)^3$

In Exercises 5–8, determine the open intervals on which the function is increasing or decreasing. Solve the problem analytically and graphically.

5. $f(x) = x^2 + x - 2$

6. $g(x) = -x^2 + 7x - 12$

7. $h(x) = \dfrac{x^2 - 3x - 4}{x - 3}$

8. $f(x) = -x^3 + 6x^2 - 2$

9. ***Meteorology*** The monthly normal temperature T (in degrees Fahrenheit) for New York City can be modeled by

$$T = 0.0385t^4 - 1.122t^3 + 9.67t^2 - 21.8t + 47$$

where $1 \le t \le 12$ and $t = 1$ corresponds to January. *(Source: National Climatic Data Center)*

(a) Find the interval(s) on which the model is increasing.

(b) Find the interval(s) on which the model is decreasing.

(c) Interpret the results of parts (a) and (b).

(d) Use a graphing utility to graph the model.

10. ***CD Shipments*** The number S of manufacturer unit shipments (in millions) of CDs in the United States from 1998 through 2002 can be modeled by

$$S = 5.8583t^3 - 28.943t^2 - 34.36t + 940.6$$

where $-2 \le t \le 2$ and $t = 0$ corresponds to 2000. *(Source: Recording Industry Association of America)*

(a) Find the interval(s) on which the model is increasing.

(b) Find the interval(s) on which the model is decreasing.

(c) Interpret the results of parts (a) and (b).

(d) Use a graphing utility to graph the model.

In Exercises 11–20, use the First-Derivative Test to find the relative extrema of the function. Then use a graphing utility to verify your result.

11. $f(x) = 4x^3 - 6x^2 - 2$

12. $f(x) = \frac{1}{4}x^4 - 8x$

13. $g(x) = x^2 - 16x + 12$

14. $h(x) = 4 + 10x - x^2$

15. $h(x) = 2x^2 - x^4$

16. $s(x) = x^4 - 8x^2 + 3$

17. $f(x) = \dfrac{6}{x^2 + 1}$

18. $f(x) = \dfrac{2}{x^2 - 1}$

19. $h(x) = \dfrac{x^2}{x - 2}$

20. $g(x) = x - 6\sqrt{x}, \quad x > 0$

In Exercises 21–30, find the absolute extrema of the function on the closed interval. Then use a graphing utility to confirm your result.

21. $f(x) = x^2 + 5x + 6; \quad [-3, 0]$

22. $f(x) = x^4 - 2x^3; \quad [0, 2]$

23. $f(x) = x^3 - 12x + 1; \quad [-4, 4]$

24. $f(x) = x^3 + 2x^2 - 3x + 4; \quad [-3, 2]$

25. $f(x) = 4\sqrt{x} - x^2; \quad [0, 3]$

26. $f(x) = 2\sqrt{x} - x; \quad [0, 9]$

27. $f(x) = 3x^4 - 6x^2 + 2; \quad [0, 2]$

28. $f(x) = -x^4 + x^2 + 2; \quad [0, 2]$

29. $f(x) = \dfrac{2x}{x^2 + 1}; \quad [-1, 2]$

30. $f(x) = \dfrac{8}{x} + x; \quad [1, 4]$

31. ***Surface Area*** A right circular cylinder of radius r and height h has a volume of 25 cubic inches. The total surface area of the cylinder in terms of r is given by

$$S = 2\pi r\left(r + \frac{25}{\pi r^2}\right).$$

Use a graphing utility to graph S and S' and find the value of r that yields the minimum surface area.

32. ***Environment*** When organic waste is dumped into a pond, the decomposition of the waste consumes oxygen. A model for the oxygen level O (where 1 is the normal level) of a pond as waste material oxidizes is

$$O = \frac{t^2 - t + 1}{t^2 + 1}, \quad 0 \le t$$

where t is the time in weeks.

(a) When is the oxygen level lowest? What is this level?

(b) When is the oxygen level highest? What is this level?

(c) Describe the oxygen level as t increases.

In Exercises 33–
graph of the fun
Then use a graph

33. $f(x) = (x -$

34. $h(x) = x^5 -$

35. $g(x) = \frac{1}{4}(-x$

36. $h(x) = x^3 -$

In Exercises 37–
the function.

37. $f(x) = \frac{1}{2}x^4 -$

38. $f(x) = \frac{1}{4}x^4 -$

39. $f(x) = x^3(x$

40. $f(x) = (x +$

In Exercises 41–
relative extrema

41. $f(x) = x^5 -$

42. $f(x) = x(x^2$

43. $f(x) = (x -$

44. $f(x) = (x -$

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45. $R = \frac{1}{1500}(15$

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47. ***Minimum***
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49. ***Newspaper***
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time peri

(b) Find the
greatest r

(c) Briefly ex