Math 16C, Winter 2020
Mar. 4, 2020

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ______________________

ID#: ______________________

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. *You must show all your work for full credit.* Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. The proctors have been directed not to answer any interpretation questions.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

1. Find and classify all critical points of the function

$$f(x,y) = x^2y - 2xy - \frac{1}{2}y^2.$$

$f_x = 2xy - 2y = 2y(x-1) = 0$ $\quad$ $\underline{x=1 \text{ or } y=0}$

$f_y = x^2 - 2x - y = 0$

When $x=1$, $y^2 = x^2 - 2x = -1$ $\quad$ $\underline{(1,-1)}$

When $y=0$, $x^2 - 2x = 0$, $x(x-2) = 0$, $x=0$, $x=2$

$\underline{(0,0)}$, $\underline{(2,0)}$

	$(0,0)$	$(2,0)$	$(1,-1)$
$f_{xx} = 2y$	0	0	-2
$f_{yy} = -1$	-1	-1	-1
$f_{xy} = 2x-2$	-2	2	0
$d = f_{xx} \cdot f_{yy} - (f_{xy})^2$	-4	-4	2
	saddle	saddle	local max.

2. Use Lagrange multipliers to find the maximum and the minimum of the function $f(x, y) = xy$ on the ellipse $4x^2 + 9y^2 = 32$.

$$F = xy - \lambda\left(4x^2 + 9y^2 - 32\right)$$

$$F_x = y - 8x\lambda = 0 \quad \cdot x \quad 8x^2\cancel{\lambda} = 18y^2\cancel{\lambda} \quad (\lambda \neq 0)$$

$$F_y = x - 18y\lambda = 0 \quad \cdot y \quad \underline{4x^2 = 9y^2}$$

$$F_\lambda = -\left(4x^2 + 9y^2 - 32\right) = 0$$

$$4x^2 + 4x^2 = 32$$

$$8x^2 = 32, \quad x^2 = 4, \quad x = \pm 2$$

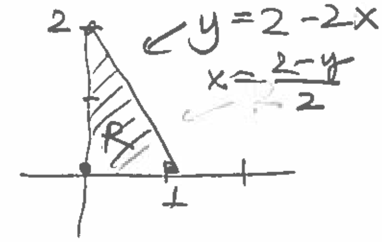
$$y^2 = \frac{4}{9}x^2 = \frac{16}{9}, \quad y = \pm\frac{4}{3}$$

Four c.p.'s:

	xy	
$(-2, -\frac{4}{3})$	$8/3$	$\leftarrow$ max
$(-2, \frac{4}{3})$	$-8/3$	$\leftarrow$ min
$(2, -\frac{4}{3})$	$-8/3$	
$(2, \frac{4}{3})$	$8/3$	

3. Consider the function $f(x, y) = x^2 y$.

(a) Find the volume of the solid which is bounded above by the surface $z = f(x, y)$, and below by the triangle R in the xy plane with vertices $(0, 0)$, $(0, 2)$ and $(1, 0)$. (Write the double integral in both orders, but evaluate only in one order.)



$$\iint_R f \, dA$$

$$= \int_0^1 \left[\int_0^{2-2x} x^2 y \quad dy \right] dx$$

$$= \int_0^2 \left[\int_0^{\frac{2-y}{2}} x^2 y \, dx \right] dy$$

$$= \int_0^1 x^2 \cdot \frac{y^2}{2} \Big|_{y=0}^{y=2-2x} dx$$

$$= 2 \int_0^1 x^2 (1-x)^2 \, dx = 2 \int_0^1 \left[x^2 - 2x^3 + x^4 \right] dx$$

$$= 2 \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = 2 \left[\frac{1}{5} - \frac{1}{6} \right] = \underline{\underline{\frac{1}{15}}}$$

(a) Find the average value of the function $z = f(x, y)$ on the triangle R. from (a).

$$\frac{\iint_R f \, dA}{\text{area of } (R)} = \underline{\underline{\frac{1}{15}}}$$

area of $(R) = 1$

4.

(a) Compute: $\sum_{n=2}^{\infty} \frac{3^{n-2}5^{-n}}{2^n}$.

$$= \sum_{n=2}^{\infty} \frac{1}{9}\left(\frac{3}{10}\right)^n$$

$$= \frac{1}{9}\cdot\left(\frac{3}{10}\right)^2 \cdot \frac{1}{1-\frac{3}{10}}$$

$$= \frac{1}{10}\cdot\frac{1}{7} = \underline{\underline{\frac{1}{70}}}$$

(b) Determine whether the sequence given by $a_n = \frac{2^n+3^n+4}{2^{n+1}+3^{n+1}+5}$ converges or not, and, if it converges, compute its limit.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n + 1 + \frac{4}{3^n}}{2\left(\frac{2}{3}\right)^n + 3 + \frac{5}{3^n}} = \underline{\underline{\frac{1}{3}}},$$