Math 16C, Winter 2020.

Practice Midterm 1

Note. These problems are a practice exam for Midterm 1. They are a bit longer than the actual exam, which will have only four problems. Give yourself 60 minutes and no distractions to write out the solutions.

1. Find the general solution to each of the following differential equations. You can leave the solutions in the implicit form.

(a) $yy' - 2xe^x = 0$ (b) $xy' + y = x^2 + 1$ (Assume x > 0.)

2. Find the function y = f(x) such that the point (1,5) lies on its graph and it satisfies the following differential equation:

$$x^2y' - 2xy = 12.$$

3. A sphere is given by the equation

$$x^{2} + y^{2} + z^{2} + 2x + 4y - 4z + 2 = 0.$$

(a) Does the point (1, 1, 1) lie inside or outside the sphere?

(b) For each of the three coordinate axes (the x-axis, the y-axis and the z-axis), determine whether the sphere intersects it.

4. An 120 gallon container contains a mixture of 30% acid and 70% water. A mixture of 80% acid and 20% water is added at the rate of 3 gallons per minute, and the tank is drained at the same rate. (a) Assuming that the liquid in the container is well-mixed, determine the time when the concentration of acid is 50%.

(b) Determine the limit of the amount of acid in the container as time goes to infinity.

5. Determine the equation of the sphere which has a diameter with endpoints (0,0,0) and (1,2,-2),

6. Consider the function $z = \sqrt{x^2 + y^2 - 1}$. Sketch the level curves for the following values of z: 0, $\sqrt{3}$, $\sqrt{8}$, $\sqrt{15}$.

7. The rate at which the number of caribou A changes at time t is proportional to $\sqrt{A + 100}$. Initially, there are 44 caribou, and after 3 years there are 125 caribou. Find the number of caribou after 4 years.