Math 16C, Winter 2020.

Practice Midterm 2

Note. These problems are a practice exam for Midterm 2. They are a bit longer than the actual exam, which will have only four problems. Give yourself 60 minutes and no distractions to write out the solutions.

1. Find and classify all the critical points of the function

$$f(x,y) = \frac{1}{8}x^4 + y^2 - x^2 - 2y.$$

2. Use Lagrange multipliers (or any other correct method) to find the maximum of $f(x,y) = e^{xy}$ on the circle $x^2 + y^2 = 8$.

3. (a) Evaluate the following double integral (sketch the region and switch the order of integration, if necessary):

$$\int_0^9 \int_{\sqrt{x}}^3 \cos(y^3)\, dy\, dx.$$

(b) Find the volume of the solid which is bounded above by the surface $z = x^2 + 3y^2$ and below by the triangle in the xy plane with vertices $(0,0)$, $(0,2)$ and $(1,2)$.

4.
(a) Compute: $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}3^{-n}}{5^n}$.

(b) Determine whether the sequence given by $a_n = \dfrac{5\cdot 3^n + 5^n}{2^n + 6\cdot 5^n}$ converges or not.

5. Find the average value of the function $f(x,y) = \cos(3x - y)$ on the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$.

6. Consider the function $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. Find and classify all critical points of this function.

7. Use Lagrange multipliers to find the minimum of the function $S = x^2 + 2y^2 + 3z^2$ on the plane $3x - 2y + z = 6$.