Math 21A–C, Fall 2005.
Nov. 23, 2005.

MIDTERM EXAM 2

NAME(print in CAPITAL letters, *first name first*): KEY

NAME(sign): ________________________

ID#: __________________

Instructions: Each of the four problems is worth 25 points. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor in determining credit. Calculators, books or notes are not allowed.

Make sure that you have a total of 5 pages (including this one) with 4 problems. Read through the entire exam before beginning to work.

1	
2	
3	
4	
TOTAL	

1.(a) Compute the derivative of $y = \sqrt{1+\sqrt{\sin x}}$. *Do not simplify!*

$$y' = \frac{1}{2}\left(1+\sqrt{\sin x}\right)^{-1/2} \cdot \frac{1}{2}(\sin x)^{-1/2} \cdot \cos x$$

(b) Find the slope of the tangent line to the curve $x^2 + y^3 + xy = 7$ at the point $(2, 1)$.

$$2x + 3y^2 \frac{dy}{dx} + y + x \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x+y}{3y^2+x}$$

Plug in $x = 2$, $y = 1$, to get

$$-\frac{4+1}{3+2} = \underline{\underline{-1}}$$

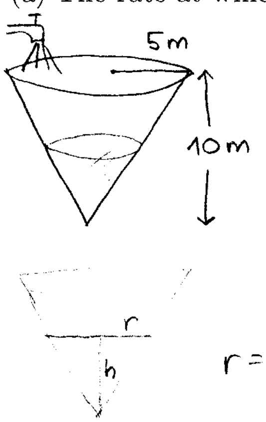
(c) Compute the linearization of $f(x) = x + \ln(x^2 - 3)$ at $x = 2$. $f(2) = 2$

$$f'(x) = 1 + \frac{1}{x^2-3} \cdot 2x \; ; \quad f'(2) = 5$$

$$L(x) = \underline{\underline{2 + 5(x-2) = 5x - 8}}$$

2. Water flows into a conical reservoir as shown in the picture. At one point, the water level in the reservoir is measured to be 4 m, and the rate of inflow of water is measured to be 2 m^3/sec. Find the following two rates. (The volume of a cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the circular base of the cone and h is its height.)

(a) The rate at which the water level in the reservoir is increasing.



$r = \frac{1}{2}h$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

Plug in $h = 4$, $\frac{dV}{dt} = 2$

$$2 = \frac{1}{4}\pi \cdot 16 \cdot \frac{dh}{dt}$$

$$\underline{\underline{\frac{dh}{dt} = \frac{1}{2\pi}}} \quad (\text{m/sec})$$

(b) The rate at which the area of the water surface in the reservoir is increasing.

$$A = \pi r^2 = \frac{\pi}{4} h^2$$

$$\frac{dA}{dt} = \frac{\pi}{4} 2h \frac{dh}{dt} = \frac{\pi}{2} h \frac{dh}{dt}$$

Plug in $h=4$, $\frac{dh}{dt} = \frac{1}{2\pi}$ to get

$$\underline{\underline{\frac{dA}{dt} = 1}} \quad (m^2/\text{sec})$$

3. At time t, the position (x, y) of a particle moving on a plane is given by $x = t^2 - t$, $y = t^2 - 2t$.
(a) Find the tangent to the curve on which the particle is moving at $t = 3$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-2}{2t-1}$$

At $t=3$, slope $= \frac{4}{5}$, point : $(6, 3)$

Tangent : $y - 3 = \frac{4}{5}(x-6)$; $y = \frac{4}{5}x - \frac{9}{5}$

(b) Find the time t at which the tangent to the curve is perpendicular to the line $y = -\frac{1}{3}x + 17$.

Slope of the tangent $= 3$

$$\frac{2t-2}{2t-1} = 3$$

$$2t - 2 = 3(2t-1)$$
$$2t - 2 = 6t - 3$$
$$4t = 1$$
$$t = \frac{1}{4}$$

4. Consider the function $f(x) = 20x^3 - 3x^5$.
(a) Is this function odd or even (or is it neither of the two)?

Odd: $f(-x) = 20(-x)^3 - 3(-x)^5 = -(20x^3 - 3x^5) = -f(x)$

(b) Determine all critical points of this function.

$f'(x) = 60x^2 - 15x^4 = 15x^2(4 - x^2)$

$x = 0, 2, -2$

(c) Determine the absolute maximum and the absolute minimum of this function on $[-1, 3]$. (Help with calculations: $20 \cdot 3^3 - 3^6 = -189$.) Then do the same for the interval $[-3, 1]$.

x	$f(x)$
-1	-17
0	0
2	$20 \cdot 8 - 3 \cdot 32 = 64$ ← max
3	-189 ← min

As the function is odd, the max. on $[-1,3]$ is 189, and the min. there is -64.